Functions — Answer sheet

Team:

Referee:

F1

Consider the function f_1 from $\{1, \ldots, 7\} \times \{1, \ldots, 7\}$ to the positive integers.

Inputs 1:										
Inputs 2:										
Outputs:										

Description:

F2

Consider the function f_2 from $\{1, \ldots, 7\} \times \{1, \ldots, 7\}$ to the positive integers.

Inputs 1:										
Inputs 2:										
Outputs:										

Description:

F3

Consider the function f_3 from $\{1, \ldots, 100\}$ to the positive integers.

Inputs:										
Outputs:										

Description:

F4

Consider the function f_4 from $\{1, \ldots, 100\}$ to the positive integers.

Inputs:										
Outputs:										

Description:

F5

Consider the function f_5 from $\{1, \ldots, 100\}$ to the positive integers.

Inputs:										
Outputs:										

Description:

Shuttle — A1 and A3

A1

The polynomial $1 - x + x^2 - x^3 + \cdots - x^9 + x^{10}$ may be written in the form $a_0 + a_1 y + a_2 y^2 + \cdots + a_9 y^9 + a_{10} y^{10}$, where $y = x + 1$ and the a_i's are constants. Find the value of a_8.

Pass on your answer to A1 as X.

A3

Y is the number you will receive.

Find the number of integers a such that $1 < a < Y$ and $n^a - n$ is divisible by 21 for all positive integers n.

Pass on your answer to A3 as Z.

Shuttle — A2 and A4

A2

X is the number you will receive.

Except for the first two terms, each term of the sequence $X, Y, X - Y, \ldots$ is obtained by subtracting the previous term from the term before that. Find the integer Y such that the first negative term in this sequence occurs as late as possible.

Pass on Y as your answer to A2.

A4

Z is the number you will receive.

An artist hangs his 2-metre-wide artwork on a wall so that the edge of the artwork touches a corner in the wall. Z art surveyors are viewing the artwork 4 metres from the wall. However, due to COVID restrictions, the art surveyors are also standing 2 metres apart from each other. Find, in degrees, the maximum sum of the viewing angles each surveyor can get.

Pass on your answer to A4.

Shuttle — B1 and B3

B1

The sum of the terms of an infinite geometric series is 2 and the sum of squares of the terms is 6. The sum of the cubes of the terms can be written as $\frac{m}{n}$ where m, n are relatively prime positive integers. Find $m + n$.

Pass on your answer to B1 as X.

B3

Y is the number you will receive.

In triangle ABC, $AB = Y$, $BC = Y + 1$, and $CA = Y + 2$. Distinct points D, E, and F lie on segments $\overline{BC}$, $\overline{CA}$, and $\overline{DE}$, respectively, such that $\overline{AD} \perp \overline{BC}$, $\overline{DE} \perp \overline{AC}$, and $\overline{AF} \perp \overline{BF}$. The length of segment $\overline{DF}$ can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $40m + 10n$?

Pass on your answer to B3 as Z.

Shuttle — B2 and B4

B2

X is the number you will receive.

Find the smallest odd prime factor of $X^7 + 1$.

Pass on your answer to B2 as Y.

B4

Z is the number you will receive.

Consider an equilateral triangle with side Z. Suppose that one move consists of changing the length of any of the sides of a triangle such that the result will still be a triangle. Find the minimum number of moves to change the given triangle to an equilateral triangle with side 2..

Pass on your answer to B4.

Shuttle — Answer sheet A

Team:

Referee:

A1				4 3 0
A2				4 3 0
A3				4 3 0
A4				4 3 0

Time: 2 1 0

Final Score: / 18

Shuttle — Answer sheet B

Team:

Referee:

B1				4 3 0
B2				4 3 0
B3				4 3 0
B4				4 3 0

Time: 2 1 0

Final Score: / 18

Relay — R1

Team: ______________________

Let $d(n)$ denote the number of digits of n in base 10. Find $d(2^{420}) + d(5^{420})$.

First attempt

Second attempt

Relay — R2

Team: ______________________

Let $a_1 = \frac{1}{2}, a_2 = \frac{1}{\sqrt{2}}$, and for $n > 2$,

$$a_n = a_{n-1}\sqrt{1 - a_{n-2}^2} - a_{n-2}\sqrt{1 - a_{n-1}^2}$$

Find a_{2022}.

First attempt

Second attempt

Relay — R3

Team: ____________________

Let S be the collection of all possible subsets of $\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots, \frac{1}{2022}\}$. Then, if A is a set of rational numbers, the function $f(A)$ returns the product of all the elements of A, where the empty set has product 1. What is the average value of f over all elements of S?

First attempt

Second attempt

Relay — R4

Team: ____________________

How many integer-sided triangles (up to congruency) have area 999/2?

First attempt

Second attempt

RELAY — R5

Team: ______________________

The numbers $2, 4, 8, \ldots, 2^{2022}$ are placed randomly in a 6×337 grid. Let R_i be the sum of the i^{th} row, and C_j be the sum of the j^{th} column. What is the probability that the R_i and C_j are both in strictly increasing order?

First attempt	Second attempt

RELAY — R6

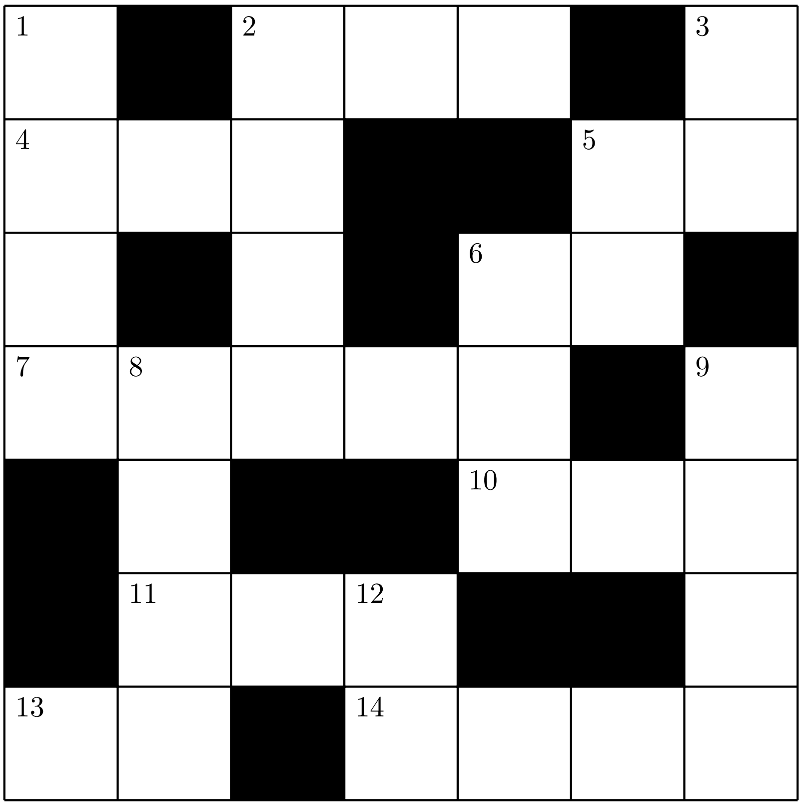
Team: ______________________

Find

$$\sum_{0 \le n-2022 \le k \le 2022} \binom{n}{k}.$$

First attempt	Second attempt

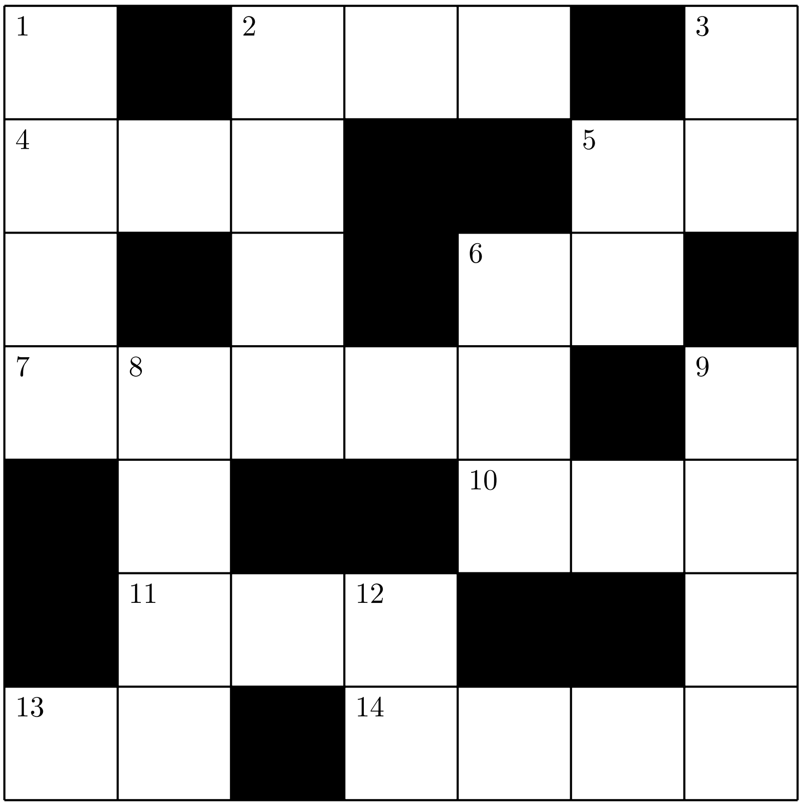
Crossnumber — Across



Across

2. The number of positive integers that divide 10^{10}, 12^{12} or 15^{15}.
4. The number of 8-digit numbers with at most 2 distinct digits such that the first and third digits are 5.
5. A number whose last digit is the square of its first digit.
6. The number of integral solutions to $x^2 + y^2 = 221$.
7. The value of $(6\tan(x))^4$ when $(6\cos(x))^3 = (6\sin(x))^2$, $0 < x < \frac{\pi}{2}$.
10. The dimension of the space of 29×29 symmetric matrices with zeros on the anti-diagonal.
11. A prime number of the form $p = 2^{2^n} + 1$.
13. The difference between 5 Down and 12 Down.
14. A number whose sequence of digits is decreasing by 2.

Crossnumber — Down



Down

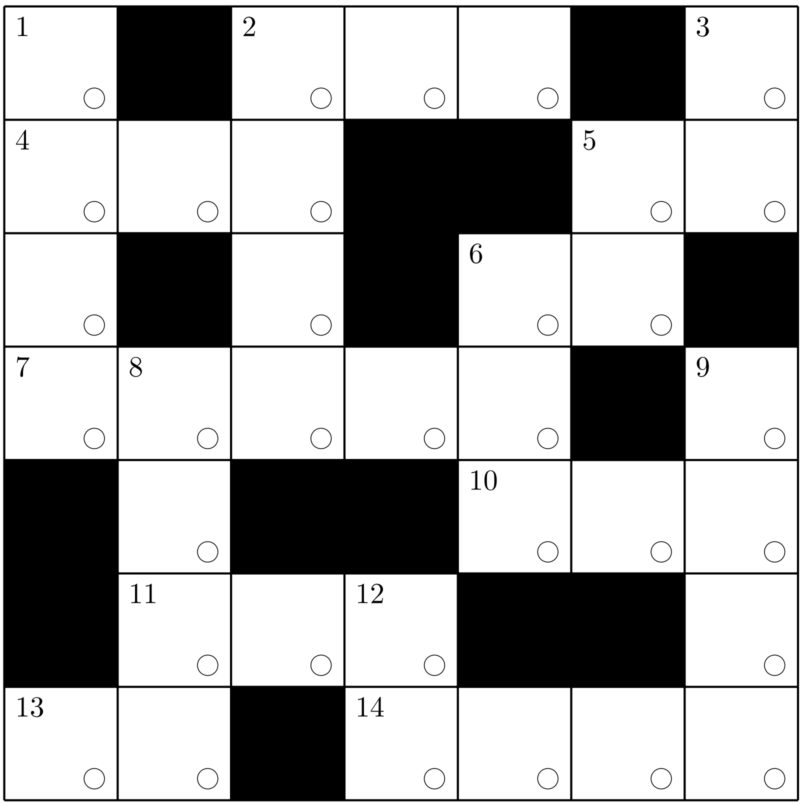
1. The largest multiple of 27 with all digits distinct and odd.
2. A third of the product of 4 Across and 5 Down.
3. The smallest $n = pq$ with p, q prime such that $(p+1)(q+1)$ reverses its digits.
5. The last two digits of 6^{2022}.
6. The volume of the region enclosed by the surfaces $x^2 + z^2 = 9$ and $y^2 + z^2 = 9$.
8. The sum of two consecutive 4th powers.
9. The integer $n < 2022$ such that $2022^3 = np + 1$, p prime.
12. The number whose digits do not appear elsewhere on this crossnumber.

Crossnumber — Answer sheet

Team:

Referee:

Totals



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/5

/4

/6

/4

/4

/6

/34