Functions — Answer sheet

Team:

Referee:

F1

Consider the function f_1 from $\{2, \ldots, 10\} \times \{2, \ldots, 10\}$ to the positive integers.

Inputs 1:										
Inputs 2:										
Outputs:										

Description:

F2

Consider the function f_2 from $\{1, \ldots, 10\} \times \{1, \ldots, 10\}$ to the non-negative integers.

Inputs 1:										
Inputs 2:										
Outputs:										

Description:

F3

Consider the function f_3 from $\{10, \ldots, 20\} \times \{10, \ldots, 20\}$ to the positive integers.

Inputs 1:										
Inputs 2:										
Outputs:										

Description:

F4

Consider the function f_4 from $\{1, \ldots, 30\}$ to the positive integers.

Inputs:										
Outputs:										

Description:

F5

Consider the function f_5 from $\{3, \ldots, 26\}$ to the positive integers.

Inputs:										
Outputs:										

Description:

Shuttle — A1 and A3

A1

Alice and Bob play Tic-Tac-Toe on a 3×3 grid, with Alice going first. If both players play randomly, the probability of a draw is $\frac{m}{n}$, where m, n are coprime positive integers. Find the value of $100m + n$.

Pass on your answer to A1 as X.

A3

Y is the number you will receive.

Triangle $\triangle ABC$ has circumcircle Ω. A smaller circle ω is tangent to Ω at point A, and also tangent to segment BC at point D. If $|AD| = 40$, $|BC| = Y$, and ω has radius 25, find $|BD| \cdot |CD|$.

Pass on your answer to A3 as Z.

Shuttle — A2 and A4

A2

X is the number you will receive.

It is raining cats, dogs, chickens, and octopuses at the rate of X animals per second, or 2560 limbs per second, or 1001 hearts per second (octopuses have 3 hearts). If dogs rain at twice the rate of cats, find the rate of raining cats (per second).

Pass on your answer to A2 as Y.

A4

Z is the number you will receive.

The Fibonacci sequence is $F_1 = F_2 = 1$, $F_{n+2} = F_{n+1} + F_n$ for all $n \geq 2$. Determine the last digit of F_Z.

Shuttle — B1 and B3

B1

A pasture of grass grows at a constant rate (independent of the amount of grass left). If the pasture can feed 19 cows over 5 days, or 15 cows over 7 days, how many cows can it feed over 10 days?

Pass on your answer to B1 as X.

B3

Y is the number you will receive.

Find the Y-th smallest positive integer that can be expressed as both $m^2 - 2$ and $3n^2 - 5$ for integers m, n.

Pass on your answer to B3 as Z.

Shuttle — B2 and B4

B2

X is the number you will receive.

Find the area of a regular X-sided polygon inscribed in the unit circle.

Pass on your answer to B2 as Y.

B4

Z is the number you will receive.

Find the smallest positive integer N such that there are at least Z integer lattice points in the region $\{0 \leq x \leq N, |y| \leq \sqrt{7x}\}$ of the Euclidean plane.

Shuttle — Answer sheet A

Team:

Referee:

A1				4 3 0
A2				4 3 0
A3				4 3 0
A4				4 3 0

Time: 2 1 0

Final Score: / 18

Shuttle — Answer sheet B

Team:

Referee:

B1				4	3 0
B2				4	3 0
B3				4	3 0
B4				4	3 0

Time: 2 1 0

Final Score: / 18

Relay — R1

Team: _____

Let ABC be a triangle with $AC = 3, BC = 4, AB = 5$. Let M be the midpoint of AB, and I be the incentre of ABC. Find $IC \cdot MC$.

First attempt

Second attempt

Relay — R2

Team: _____

Let $\varphi(n)$ denote the number of positive integers less than n that are coprime to n. Find the first m for which

$$\varphi^m(20^{23}) = 1.$$

First attempt

Second attempt

Relay — R3

Team: _____

Let S be the set of positive integers whose prime factors are all less than 10. Find

$$\sum_{x \in S} \frac{1}{x}.$$

First attempt

Second attempt

Relay — R4

Team: _____

11 numbers are drawn randomly without replacement from the set $\{1, 2, \ldots, 1001\}$. Let X be this set of numbers, and Y be the number of elements in X ending in 1. Find $\mathbb{E}[Y^2]$.

First attempt

Second attempt

Relay — R5

Team: _____

Find the remainder when

$$\underbrace{(((\ldots(}_{2023!-1}20^5+23)^5+23)^5+\ldots)^5+23)^5$$

is divided by 2023.

First attempt

Second attempt

Relay — R6

Team: _____

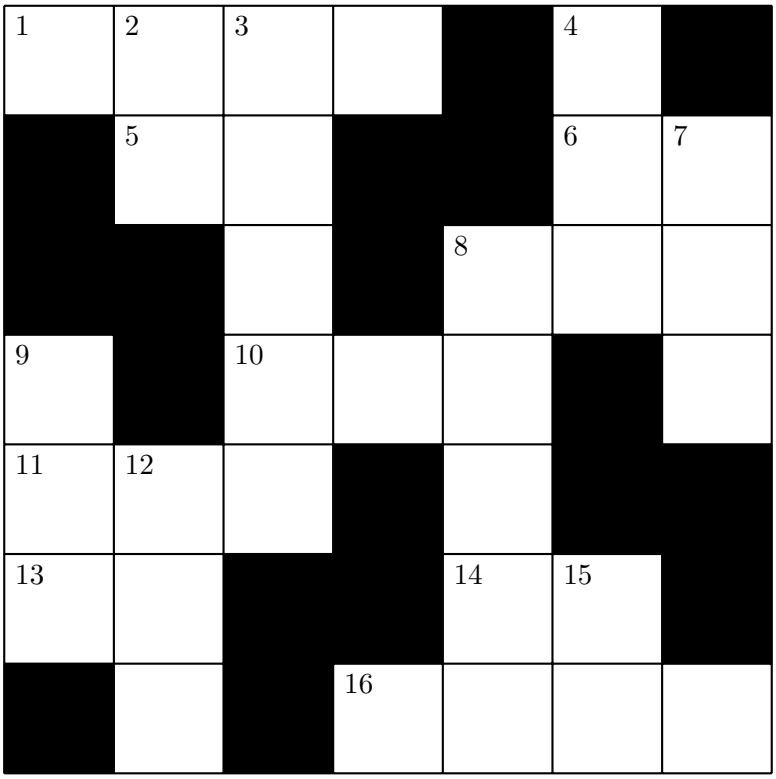
Find the number of non-empty subsets S of $\{1, 2, \ldots, 2023\}$ such that for all $x, y \in S$,

$$|x-y| \not\equiv 2 \pmod 3.$$

First attempt

Second attempt

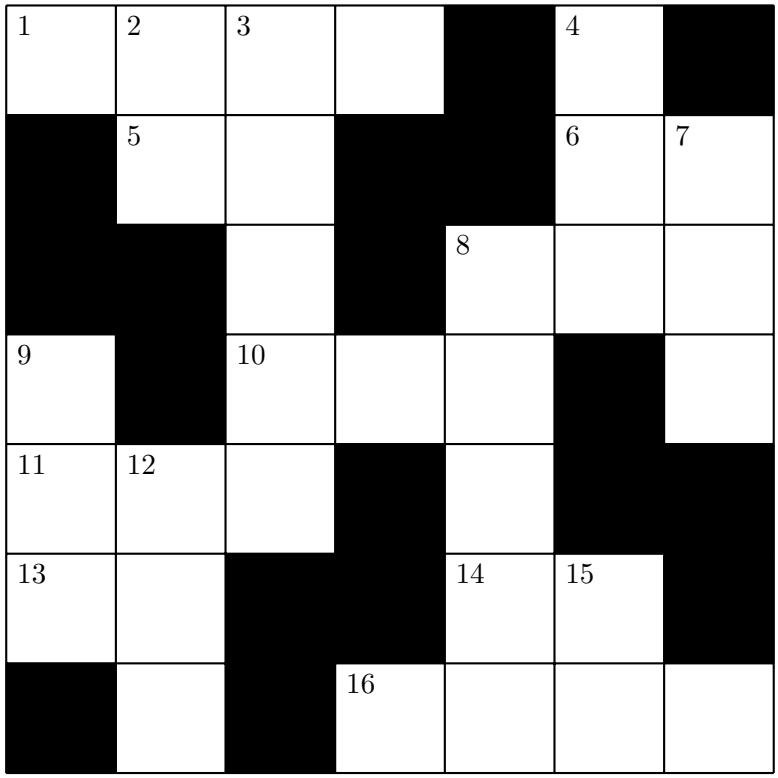
Crossnumber — Across



Across

1. $2^n - 2^{n-5} + 2^{n-10}$ (for some n)
5. Highly composite number.
6. Pentagonal number.
8. Volume of a cuboid with integral side lengths where the smallest face is square and the length of all its edges is 96.
10. The digit sum of this number is the same in base 2 and base 3.
11. John sits an exam with 4 problems. Each problem is scored as an integer from 0 to k. If his score is is strictly decreasing with each subsequent problem, how many possible ways could the scores be distributed among the 4 problems.
13. One third of 9 DOWN.
14. n where a regular n-gon has integer interior angle in degrees.
16. A palindrome.

Crossnumber — Down



Down

2. m such that $3m + 1$ and $3m - 1$ are prime.

3. A power of 5.

4. A prime whose digit sum is a prime.

7. Double of 6 ACROSS.

8. Bugs Bunny plays a game in the Euclidean plane. At the n-th minute ($n \geq 1$), Bugs Bunny hops a unit distance North or East if n is odd and South or West if n is even. After some fixed time k (for some k), we check and find that Bugs Bunny has returned to his original location. How many unique paths could Bugs Bunny have taken.

9. A multiple of $k^3 + 4$ where k is the last digit of 16 ACROSS.

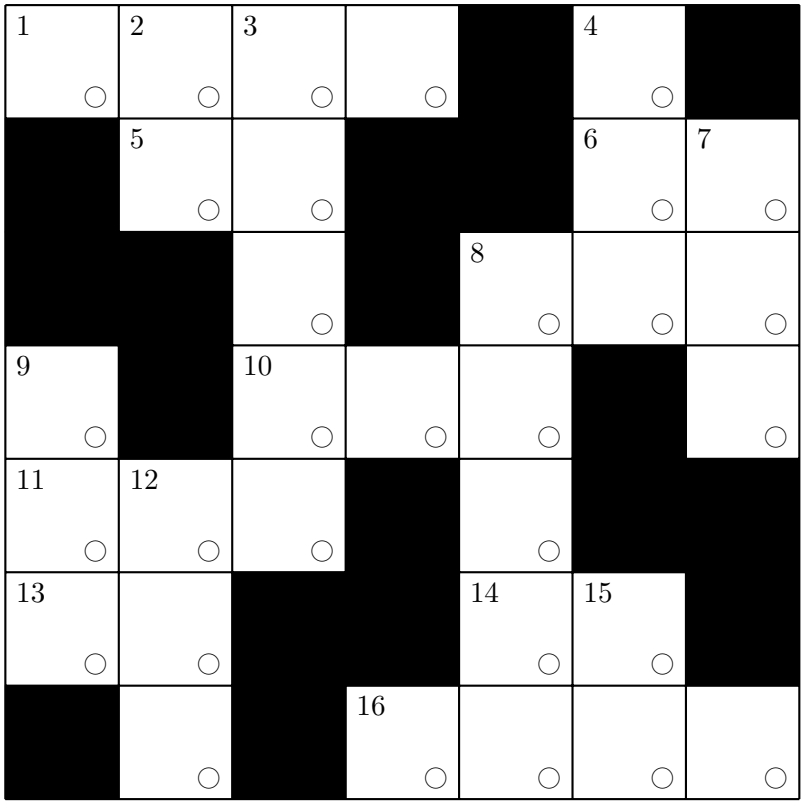
12. $2^{n-2} + 3^{n-3} + 5^{n-5}$ (for some n)

15. Volume of a cuboid with integral side lengths such that 3 times its volume is equal to twice the total length of all its edges.

Crossnumber — Answer sheet

Team:

Referee:



Totals

/5

/4

/4

/5

/4

/4

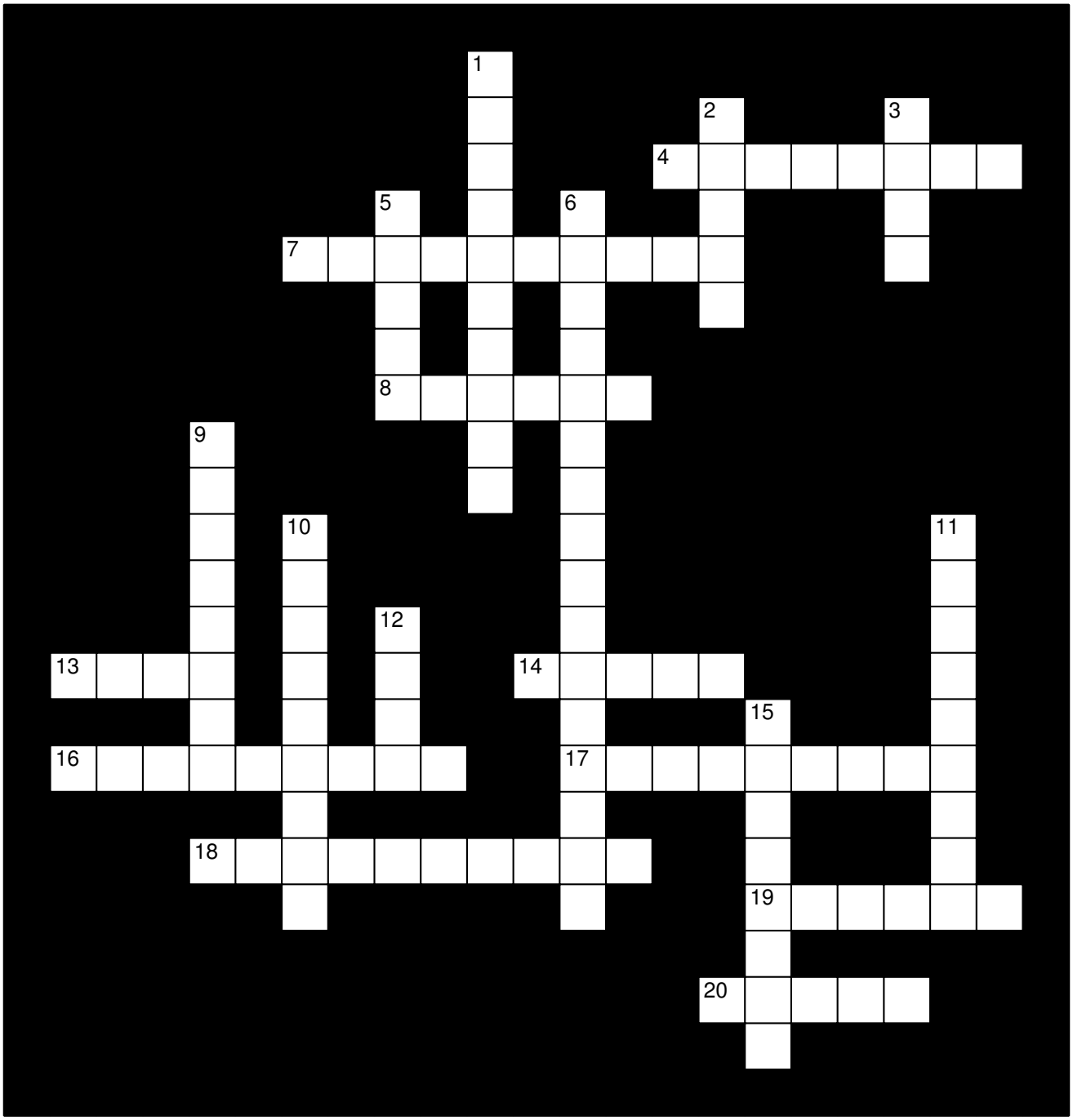
/5

/31

Quizdle — Crossword Sheet

Team:

Referee:



Quizdle — Answer Sheet

Team: _____

Referee: _____

	Across	H	M	E
4		2	$1\frac{1}{2}$	$\frac{1}{2}$
7		2	$1\frac{1}{2}$	$\frac{1}{2}$
8		2	$1\frac{1}{2}$	$\frac{1}{2}$
13		2	$1\frac{1}{2}$	$\frac{1}{2}$
14		2	$1\frac{1}{2}$	$\frac{1}{2}$
16		2	$1\frac{1}{2}$	$\frac{1}{2}$
17		2	$1\frac{1}{2}$	$\frac{1}{2}$
18		2	$1\frac{1}{2}$	$\frac{1}{2}$
19		2	$1\frac{1}{2}$	$\frac{1}{2}$
20		2	$1\frac{1}{2}$	$\frac{1}{2}$

	Down	H	M	E
1		2	$1\frac{1}{2}$	$\frac{1}{2}$
2		2	$1\frac{1}{2}$	$\frac{1}{2}$
3		2	$1\frac{1}{2}$	$\frac{1}{2}$
5		2	$1\frac{1}{2}$	$\frac{1}{2}$
6		2	$1\frac{1}{2}$	$\frac{1}{2}$
9		2	$1\frac{1}{2}$	$\frac{1}{2}$
10		2	$1\frac{1}{2}$	$\frac{1}{2}$
11		2	$1\frac{1}{2}$	$\frac{1}{2}$
12		2	$1\frac{1}{2}$	$\frac{1}{2}$
15		2	$1\frac{1}{2}$	$\frac{1}{2}$

Minor spelling mistakes are allowed and will be fixed by the marker.

Round down sum of scores to the nearest integer to get...

Final Score: _____ / 40

Quizdle — Across (Hard)

4. This word, derived from the Latin word for "small pebble". One kind of this named after Hermann Schubert is a branch of algebraic geometry introduced in the 19th century to solve various counting problems of projective geometry. It also precedes the words "of negligence" where used to denote a legal standard in U.S. tort law to determine if a duty of care has been breached.

7. This two-time gold medallist at the 1994 and 1995 International Mathematical Olympiads gained their bachelor's degree from the Sharif University of Technology and their PhD from the University of Harvard. Their thesis was *Simple geodesics on hyperbolic surfaces and the volume of the moduli space of curves.*

8. This person is the author of the books *Things to Make and Do in the Fourth Dimension* and *Humble Pi*, the latter of which was first maths book in the UK to be a Sunday Times No. 1 bestseller. They collaborated with Brady Haran to produce a calculator-unboxing YouTube video series, presenting these videos as a member of a fictional "Calculator Appreciation Society."

13. There is no standard algorithm used for this concept, though CORDIC and precomputation with linear interpolation are commonly used. Etymologically, the word derives from the Sanskrit word *jyā* meaning "bow-string" and the Arabic word *jaib*, which means "bosom", "pocket", or "fold". It was later translated into Latin as a word meaning "bay" or "fold".

14. When "ratio" is appended to this word, the result denotes a quantity associated with four collinear points A, B, C, and D with formula $\frac{AC \cdot BD}{BC \cdot AD}$. It is also known as the double ratio or the anharmonic ratio.

16. Things named after this surname include a namesake number, polynomials, process, trial, principle, triangle, grip, and inequality. The law of large numbers was originally referred to as their "Golden Theorem."

17. In group theory, a lemma by this name states the following: If G is a group with subgroups A and C, and $B \lhd A$ and $D \lhd C$, then there is an isomorphism between $(A \cap C)B/(A \cap D)B$ and $(A \cap C)D/(B \cap C)D$. It is so named due to the resemblance between this and the Hasse diagram of the subgroups involved. It is also known as Zassenhaus lemma or sometimes as the fourth isomorphism theorem.

18. The "universal" type of this concept can be understood as the right adjoint of a functor between power sets.

19. Roger and Lionel Penrose, impressed by this person's work *Relativity*, devised the Penrose triangle and the Penrose Staircase, the latter of which inspired this person's work *Ascending and Descending.*

20. This surname is the namesake of an impossibility theorem or paradox in social choice theory which states that when voters have more than two distinct options, no ranked voting electoral system can convert the ranked preferences of individuals into a community-wide ranking while also satisfying the the criteria of unrestricted domain, non-dictatorship, Pareto efficiency, and independence of irrelevant alternatives.

Quizdle — Down (Hard)

1. This person was supposedly the first person to call themselves a "philosopher." One of their most notable teachings was the "transmigration of souls," which holds that the soul is immortal and enters a new body when its previous body dies.

2. One mathematician with this first name names a theorem, together with Calyampudi Radhakrishna Rao, which characterizes the transformation of an arbitrarily crude estimator into an estimator that is optimal by the mean-squared-error criterion. They were the seventh African American to gain a Ph.D. in mathematics and the first to be inducted into the National Academy of Sciences.

3. An isomorphism known by this name is the musical isomorphism that maps the tangent bundle to the cotangent bundle. The term also denotes a subset of a matroid whose closure is itself, sometimes known as a subspace.

5. The Alhambra in Granada, Spain is remarkable in that its tiling patterns use nearly all, if not all, of the seventeen possible "wallpaper" type of this. They can be enumerated using crystallographic or orbifold notation.

6. This problem is part of the name of the first book of the *Rememberance of Earth's Past* science fiction trilogy by Liu Cixin, in which aliens from a planet orbiting Alpha Centauri plan to invade Earth.

9. Aged 12, this person discovered a novel proof of Pythagoras theorem using dissection but without needing to rearrange the dissected pieces. For their political activism during their visits to the US, the FBI kept a dossier on them, which reached 1427 pages by the time of their death.

10. If Hilbert was rated 80, then this mathematician would be rated 100. On December 22, 2012, a Google doodle celebrated the 125th anniversary of this mathematician birth.

11. This is the number of shillings in a guinea, the number of the amendment that ended Prohibition in the US, the number of trump cards in a tarot deck excluding The Fool, the number of sun rays on the Kurdistan flag, and the fifth Motzkin number.

12. A mathematician with this first name wrote the 25-page essay *A Mathematician's Lament*, also known as "Lockhart's Lament," which criticises the way that mathematics is taught in American schools and argues for an aesthetic, intuitive, and problem-oriented approach to teaching.

15. During the Japanese Edo period, members of all social classes inscribed *sangaku*, which is this kind of problem, on wooden tablets. These were then placed at Shinto shrines or Buddhist temples as offerings to the kami and buddhas.

Quizdle — Across (Medium)

4. Another kind of this named after Kiyosi Itô allows deeper analysis of stochastic processes such as Brownian motion and mathematical finance. It names branches of mathematically-related study when prefixed with lambda, kappa, rho, pi, epsilon, and mu.

7. This Iranian mathematician's thesis showed that the number of simple closed geodesics of length less than L is asymptotic to cL^{6g-6}, where g is the genus and c is a constant. They also proved that William Thurston's earthquake flow on Teichmüller space is ergodic. In 2014, with Alex Eskin, they proved that complex geodesics and their closures in moduli space are regular, rather than irregular or fractal.

8. In October 2017, this person started a petition to "Update the UK Traffic Signs Regulations to a geometrically correct football." They are also the namesake of this "magic square":

$$\begin{matrix} 29^2 & 1^2 & 47^2 \\ 41^2 & 37^2 & 1^2 \\ 23^2 & 41^2 & 29^2 \end{matrix}$$

13. Historically, this mathematical concept could be prefixed with "ver-", "cover-", and "haver-", among others, to form similar but distinct concepts. A law or rule named after this concept equates three quotients and a diameter.

14. When "product" is appended to this word, the result denotes a binary operation that only makes sense in three or seven dimensions. The operation in these dimensions can be described by the quaternion group and octonion group, respectively.

16. This is the surname of a family from Basel notable for having produced eight mathematically-gifted academics who contributed greatly to the development of mathematics and physics during the early modern period. Their first names were Jacob, Johann, Nicolaus I, Nicolaus II, Daniel, Johann II, Johann III, and Jacob II.

17. In Euclidean geometry, a theorem by this name states the following: Let M be the midpoint of a chord PQ of a circle, through which two other chords AB and CD are drawn; AD and BC intersect chord PQ at X and Y correspondingly. Then M is the midpoint of XY.

18. A formula is in prenex normal form if it is written as a prefix, which consists of a string of these and bound variables, followed by a matrix, which do not contain any of these.

19. This person is one of three people in the title of Douglas Hofstadter's 1979 Pulitzer Prize-winning book. The other two people are Gödel and Bach.

20. This word is also a noun. A variation of one of these in the shape of a lightning bolt is sometimes used to indicate a contradiction. A quiver in mathematics uses many of these.

Quizdle — Down (Medium)

1. This person founded a school in which initiates were sworn to secrecy and lived a communal, ascetic lifestyle. A tuning named after them is based on repeatedly adding or subtracting perfect fifths, and the frequency ratios of the notes of the major scale to the tonic in this tuning are 1:1, 9:8, 81:64, 4:3, 3:2, 27:16, 243:128, and 2:1.

2. The previous mathematician's last name is Blackwell. In 1920, another mathematician with this first name proposed their namesake "program," which aimed to ground all existing theories on a finite, complete set of axioms, and provide a proof that these axioms were consistent. After hearing that a student had dropped out of their class to study poetry, they famously responded "Good, he did not have enough imagination to become a mathematician."

3. A module M over a ring R is known by this term if taking the tensor product over R with M preserves exact sequences.

5. These things can be described as loops that are also monoids. Cayley's theorem states that all of these appear as substructures in their "symmetric" type.

6. In 1767, Euler found three families of solutions to this problem, and in 1772, Lagrange found another family. These four solutions are related to the five Lagrangian points. In 1993, a solution involving a figure-eight shape was discovered numerically by physicist Cris Moore.

9. This person's *annus mirabilis* in 1905 saw the publication of four groundbreaking papers, including one about Brownian motion for which they would win the Nobel Prize in 1921. They are also the namesake of the 99th element of the periodic table.

10. In *Good Will Hunting*, professor Gerald Lambeau explains to Sean Maguire the genius of Will Hunting by comparing him to this mathematician. It was said that every positive integer was one of this mathematicians personal friends.

11. This is the atomic number of Scandium, half the jersey number of Jackie Robinson, a third of the international calling code of the Phillipines, and a quarter of the number expressed in morse code as `---.. ....-`

12. Another mathematician with this first name won the Fields Medal in 1966 for proving that that the continuum hypothesis and the axiom of choice are independent from Zermelo–Fraenkel set theory. Their last name is Cohen.

15. This subject area was one of the Seven Liberal Arts grouped into the *quadrivium*, which contained three other subject areas: arithmetic, music, and astronomy.

Quizdle — Across (Easy)

4. Name this field of mathematics, developed concurrently by Isaac Newton and Gottfried Wilhelm Leibniz in the 17th century.

7. Name this female Iranian mathematician who was awarded the Fields Medal in 2014 for "her outstanding contributions to the dynamics and geometry of Riemann surfaces and their moduli spaces."

8. Name this recreational mathematician and comedian who runs the YouTube channel *Stand-up Maths*. His first name is Matt.

13. Name this periodic mathematical function, whose only fixed point is 0. It also has a complementary function, whose only fixed point is the Dottie number, equal to approximately 0.739085.

14. When "multiply" is appended to this word, the result denotes the action of transforming an equation like $\frac{a}{b} = \frac{c}{d}$ into $ad = bc$.

16. Their namesake "distribution" is a special case of the binomial distribution where the number of trials is one. Their namesake "principle" is usually expressed in the equation
$$\frac{1}{2}\rho v^2 + \rho g h + P = \text{constant}.$$

17. In chaos theory, an effect known by this name states that the sensitive dependence on initial conditions in which a small change in one state of a deterministic nonlinear system can result in large differences in a later state.

18. The "universal" type of this concept is denoted using a letter written upside-down, and the "existential" type of this concept is denoted using a letter written backwards.

19. Name this graphic artist whose artwork features explorations of infinity, symmetry, tesselations, and hyperbolic geometry, among other things. His sketch *Drawing Hands* depict two hands simultaneously drawing each other.

20. A type of notation introduced by Donald Knuth uses this symbol to notate very large numbers, and they are infamously used to define Graham's number. Mathematicians also use this symbol to represent a functor, or edges in a digraph.

Quizdle — Down (Easy)

1. A theorem named after this person is reputed to have the most proofs of any theorem in mathematics. Generalisations of this theorem include the law of cosines (or cosine rule) and Parseval's identity.

2. One of the second mathematician's most lasting contributions to mathematics were his namesake problems, published in 1900 and numbering 23 in total. He is also well-known for his namesake space.

3. _____*land: A Romance of Many Dimensions* is the title of a satirical novella by Edwin Abbott Abbott, first published in 1884. In the book, women are line segments, while men are polygons with various numbers of sides.

5. Name this abstract mathematical object, whose early researchers include Joseph Louis Lagrange, Niels Henrik Abel and Évariste Galois. The classification of the "finite simple" type of these objects was finished in 2004.

6. Give the name of the problem which, given the initial positions and velocities of three point masses, tries to determine their subsequent motion.

9. The other three papers outlined the theory of the photoelectric effect, introduced special relativity, and demonstrated mass-energy equivalence.

10. The first sentence in the Hard clue comes from a quote by G. H. Hardy. This mathematician made many contributions to mathematical analysis, number theory, infinite series, and continued fractions. His four notebooks have been analysed and studied for decades since his death as a source of new mathematical ideas.

11. This is the answer to 9 + 10 according to @DREHUPEMSU and @WESTROSECRAN, and half of the answer to 6 × 9 according to Douglas Adams.

12. A third mathematician with this first name published over 1500 papers over the course of their lifetime. Their namesake "number" measures, for each mathematician, the number of steps in the shortest path between them and the mathematician in terms of co-authorships.

15. Today, branches of this subject can be Riemannian, differential, algebraic, and discrete, among others. This word together with "Dash" is the name of a popular music platformer game released in 2013.