

# FUNCTIONS — ANSWER SHEET

Team:

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Referee:

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## F1

Consider the function  $f_1$  from  $\{2, \dots, 10\} \times \{2, \dots, 10\}$  to the non-negative integers.

Inputs 1:



Inputs 2:

Outputs:

Description:

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## F2

Consider the function  $f_2$  from  $\{2, \dots, 10\} \times \{2, \dots, 10\}$  to the positive integers.

Inputs 1:



Inputs 2:

Outputs:

Description:

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### F3

Consider the function  $f_3$  from  $\{2, \dots, 10\} \times \{2, \dots, 10\}$  to the positive integers.

Inputs 1:	<table border="1"><tr><td></td><td></td><td></td><td></td><td></td></tr></table>						<table border="1"><tr><td></td><td></td><td></td><td></td><td></td></tr></table>					
Inputs 2:	<table border="1"><tr><td></td><td></td><td></td><td></td><td></td></tr></table>						<table border="1"><tr><td></td><td></td><td></td><td></td><td></td></tr></table>					
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Description:

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### F4

Consider the function  $f_4$  from  $\{2, \dots, 10\} \times \{2, \dots, 10\}$  to the positive reals.

Inputs 1:	<table border="1"><tr><td></td><td></td><td></td><td></td><td></td></tr></table>						<table border="1"><tr><td></td><td></td><td></td><td></td><td></td></tr></table>					
Inputs 2:	<table border="1"><tr><td></td><td></td><td></td><td></td><td></td></tr></table>						<table border="1"><tr><td></td><td></td><td></td><td></td><td></td></tr></table>					
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Description:

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### F5

Consider the function  $f_5$  from  $\{1, \dots, 100\}$  to the non-negative integers.

Inputs:	<table border="1"><tr><td></td><td></td><td></td><td></td><td></td></tr></table>						<table border="1"><tr><td></td><td></td><td></td><td></td><td></td></tr></table>					
Outputs:	<table border="1"><tr><td></td><td></td><td></td><td></td><td></td></tr></table>						<table border="1"><tr><td></td><td></td><td></td><td></td><td></td></tr></table>					

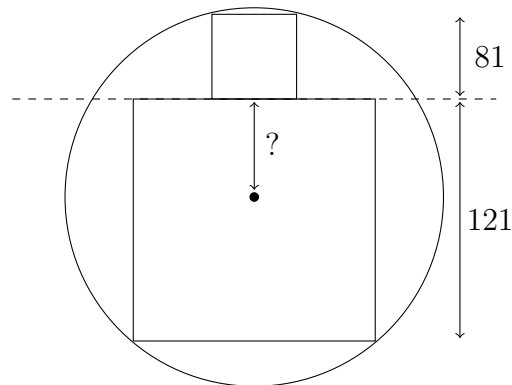
Description:

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# SHUTTLE — A1 AND A3

## A1

In the following diagram (not drawn to scale), determine the distance between the centre of the circle and the line shared by both squares (with side lengths 81 and 121 respectively).



Pass on your answer to A1 as  $X$ .

## A3

$Y$  is the number you will receive.

Find the value of

$$(1 \cdot 2)^{100! - 1} + (2 \cdot 3)^{100! - 1} + \dots + (99 \cdot 100)^{100! - 1} \pmod{Y + 100}.$$

Pass on your answer to A3 as  $Z$ .

# SHUTTLE — A2 AND A4

## A2

$X$  is the number you will receive.

A real number  $r > 0$  satisfies

$$1 + 3r + 6r^2 + 10r^3 + \cdots = X + Xr + Xr^2 + Xr^3 + \cdots .$$

If  $r = \frac{m}{n}$ , where  $m, n$  are coprime positive integers, find the value of  $100m + n$ .

Pass on your answer to A2 as  $Y$ .

## A4

$Z$  is the number you will receive.

The positive integer  $N$  satisfies

$$N = \sum_{k=0}^Z \binom{Z-k}{k} (-4)^{Z-k}.$$

Find the sum of the distinct prime factors of  $N$ .

# SHUTTLE — B1 AND B3

## B1

2024 dons are standing on the vertices of a regular 2024-gon. Some dons are wearing black caps, and some are wearing white caps. Interestingly, no don with a white cap is equidistant from two dons with black caps. Given that Tony Soprano has a white hat, find the minimum possible number of dons with white hats.

Pass on your answer to B1 as  $X$ .

## B3

$Y$  is the number you will receive.

A cone has a sphere of diameter  $Y$  internally tangent to it. A plane parallel to the base of the cone is tangent to the sphere, and cuts the cone in a circle of radius 36. What is the radius of the base of the cone?

Pass on your answer to B3 as  $Z$ .

# SHUTTLE — B2 AND B4

## B2

$X$  is the number you will receive.

Find the number of positive integers  $k$  such that  $k = \lfloor X/i \rfloor$  for some positive integer  $i$ .

Pass on your answer to B2 as  $Y$ .

## B4

$Z$  is the number you will receive.

Dylan has the tuples  $(1, 1), (2, 1), \dots, (50, 1)$  written on a blackboard. In a move, he is allowed to erase the tuples  $(a, b), (c, d)$  and write down the tuple  $(ac + Z \cdot bd, ad + bc)$ . After 49 moves, he ends up with the tuple  $(x, y)$ . Over all possible values of  $y$  he could end up with, what is the maximum value  $N$  such that  $Z^N \mid y$ ?

# SHUTTLE — ANSWER SHEET A

Team:

Referee:

**A1**    4 3 0

**A2**    4 3 0

**A3**    4 3 0

**A4**    4 3 0

Time:  2 1 0

Final Score:  / 18

# SHUTTLE — ANSWER SHEET B

Team:

Referee:

**B1**    4 3 0

**B2**    4 3 0

**B3**    4 3 0

**B4**    4 3 0

Time:  2 1 0

Final Score:  / 18



# RELAY — R1

Team: \_\_\_\_\_

What is the maximal positive integer  $N \leq 2024$  for which  $\prod_{n=1}^N n2^n$  is a multiple of 5?

First attempt

Second attempt

# RELAY — R2

Team: \_\_\_\_\_

Let  $ABC$  be a triangle with side lengths  $AB = 4, BC = 5, CA = 6$ . Let  $D$  be on the circumcircle of  $ABC$  and  $AD \perp BC$ . Let  $AD, BC$  intersect at a point  $H$ . Find  $DH$ .

First attempt

Second attempt

# RELAY — R3

Team: \_\_\_\_\_

Find

$$\sum_{n=1}^{\infty} \exp \frac{1}{n2^n} .$$

First attempt

Second attempt

# RELAY — R4

Team: \_\_\_\_\_

Two numbers are drawn randomly without replacement from the set  $\{1, 2, \dots, 100\}$ . Let  $X$  be the product of this set of numbers. Find  $E[X]$ .

First attempt

Second attempt

## RELAY — R5

Team: \_\_\_\_\_

Find the remainder when  $2^{2^{2024}}$  is divided by 99.

First attempt

Second attempt

## RELAY — R6

Team: \_\_\_\_\_

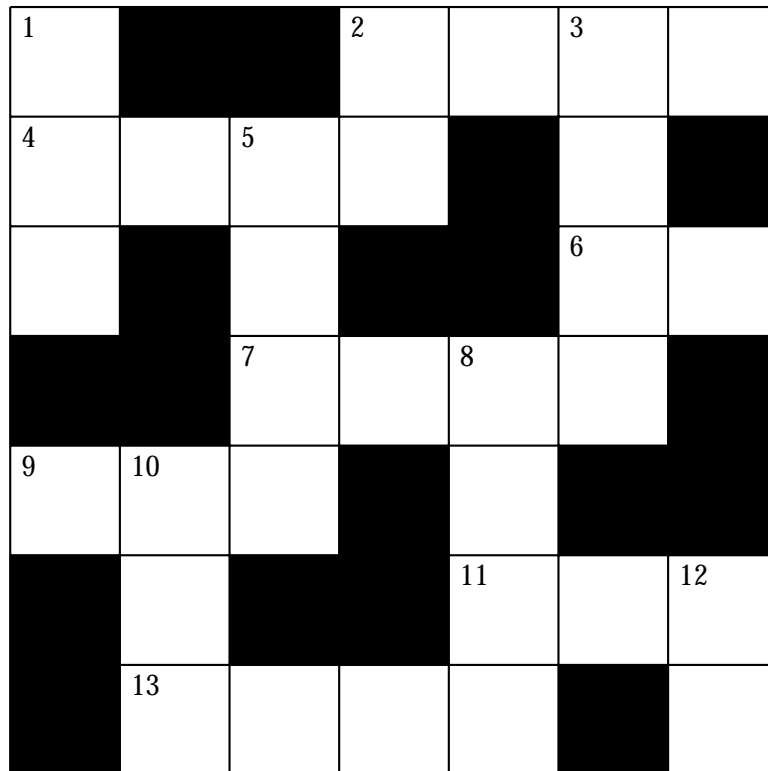
Find the number of nonempty subsets  $S$  of  $\{1, 2, \dots, 2024\}$  such that for all  $x \neq y \in S$ ,

$$4 \mid xy.$$

First attempt

Second attempt

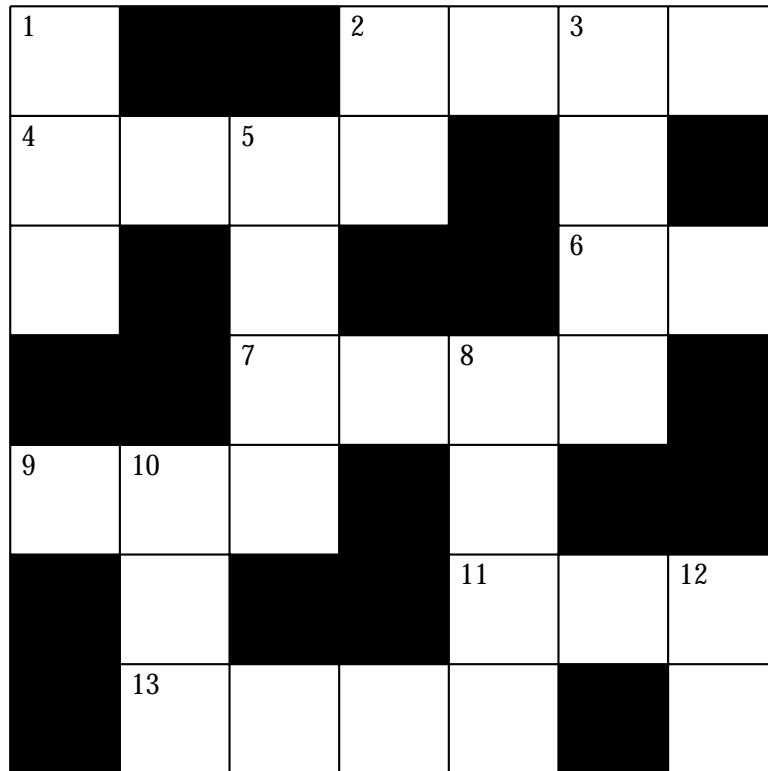
# CROSSNUMBER — ACROSS



## Across

2. A Fibonacci number.
4. The number of ways to place 2 non-attacking identical kings on a  $7 \times 7$  chessboard.
6. This number is the sum of digits of its cube.
7. 10 Down, written in base-5.
9. The square of the sum of prime divisors of 1 Down.
11. The total sum of digits in this grid.
13. The number of axis-aligned squares in a (6 Across) by (12 Down) grid.

# CROSSNUMBER — DOWN



## Down

1.  $\frac{{}^j P_{(7+P_{53})^4} k}{{}^8 P_{53}}$ .
2.  $\sec^2 \frac{2}{11} + \sec^2 \frac{2}{11} + \sec^2 \frac{3}{11} + \sec^2 \frac{4}{11} + \sec^2 \frac{5}{11}$ .
3. A number  $n$  for which each digit is used exactly once in  $3n$  and  $8n$  combined.
5. A product of 3 consecutive primes.
8. A number whose square only contains the digits 6 and 9.
10. A number that, when written in base-5, has all four digits the same.
12.  $\lceil \sin(88^\circ) + \cos(88^\circ) + \tan(88^\circ) \rceil$ .

# CROSSNUMBER — ANSWER SHEET

Team:

Referee:

Totals

1 ○			2 ○	○	3 ○	○	
4 ○	○	5 ○	○			○	
○			○			6 ○	
		7 ○	○	8 ○	○		
9 ○	10 ○	○			○		
		○			11 ○	○	12 ○
		13 ○	○	○	○		

/5

/5

/4

/4

/4

/4

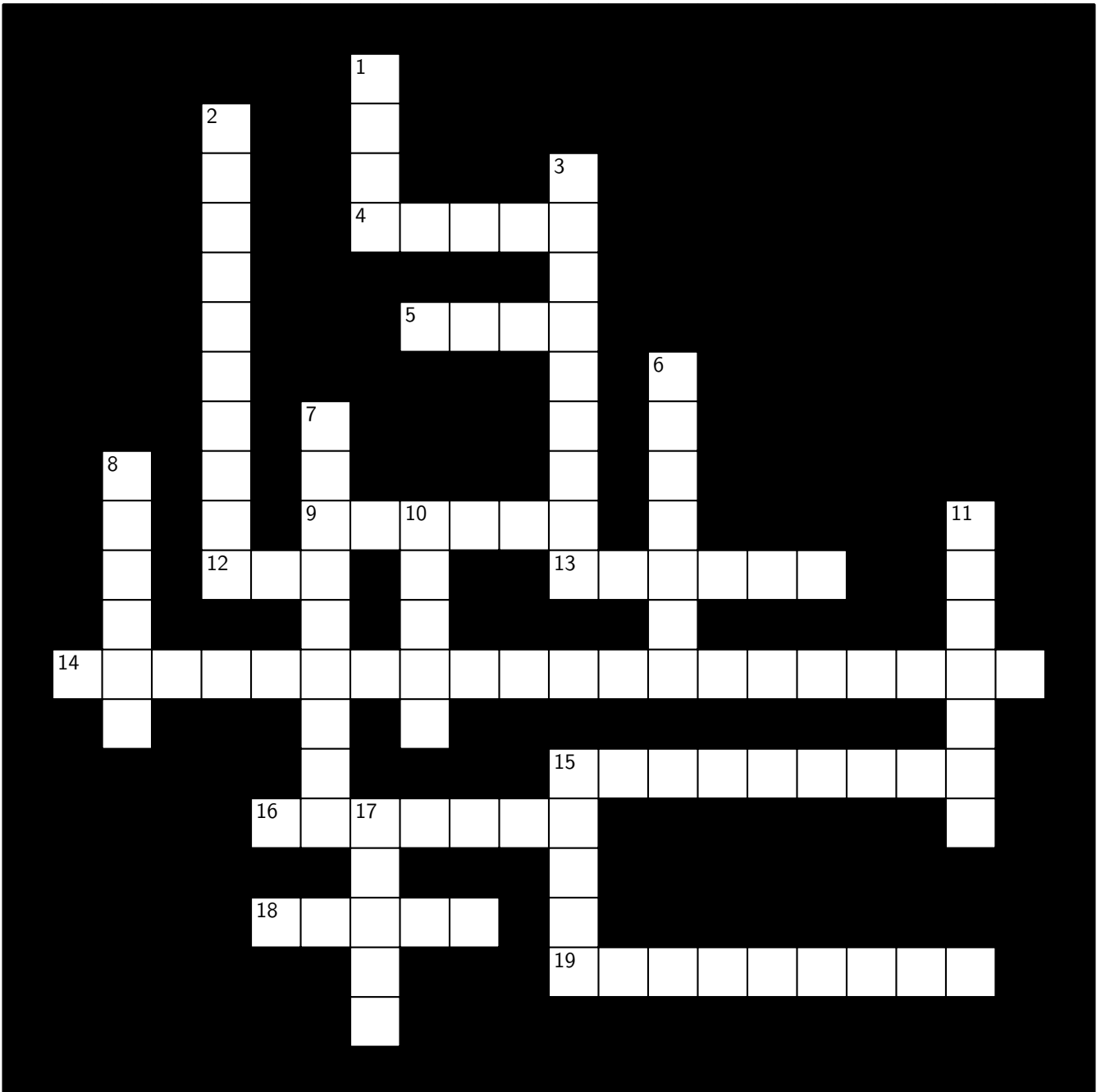
/5

/31

# QUIZDLE — CROSSWORD SHEET

Team:

Referee:



# QUIZDLE — ANSWER SHEET

Team:

Referee:

	Across	H	M	E		Down	H	M	E
4	<input type="text"/>	1	$\frac{1}{2}$	1	$\frac{1}{2}$	1	<input type="text"/>	1	$\frac{1}{2}$
5	<input type="text"/>	1	$\frac{1}{2}$	1	$\frac{1}{2}$	2	<input type="text"/>	1	$\frac{1}{2}$
9	<input type="text"/>	1	$\frac{1}{2}$	1	$\frac{1}{2}$	3	<input type="text"/>	1	$\frac{1}{2}$
12	<input type="text"/>	1	$\frac{1}{2}$	1	$\frac{1}{2}$	6	<input type="text"/>	1	$\frac{1}{2}$
13	<input type="text"/>	1	$\frac{1}{2}$	1	$\frac{1}{2}$	7	<input type="text"/>	1	$\frac{1}{2}$
14	<input type="text"/>	1	$\frac{1}{2}$	1	$\frac{1}{2}$	8	<input type="text"/>	1	$\frac{1}{2}$
15	<input type="text"/>	1	$\frac{1}{2}$	1	$\frac{1}{2}$	10	<input type="text"/>	1	$\frac{1}{2}$
16	<input type="text"/>	1	$\frac{1}{2}$	1	$\frac{1}{2}$	11	<input type="text"/>	1	$\frac{1}{2}$
18	<input type="text"/>	1	$\frac{1}{2}$	1	$\frac{1}{2}$	15	<input type="text"/>	1	$\frac{1}{2}$
19	<input type="text"/>	1	$\frac{1}{2}$	1	$\frac{1}{2}$	17	<input type="text"/>	1	$\frac{1}{2}$

Minor spelling mistakes are allowed and will be fixed by the marker.

Round down sum of scores to the nearest integer to get...

Final Score:

/ 30



# QUIZDLE — ACROSS (HARD)

4. This mathematician gives their name, along with Weinberg, to a principle in population genetics which states that allele and genotype frequencies in a population will remain constant from generation to generation in the absence of other evolutionary influences. This debunked the prevailing view that a dominant allele would automatically tend to increase in frequency. Among other things, they also have a namesake space and a namesake field.
5. The gyrobifastigium – constructed by joining two face-regular triangular prisms along corresponding square faces, giving a quarter-turn to one prism – is the only Johnson solid to be able to do this. A problem involving sets of coloured squares named after Hao Wang concerns whether the squares are able to do this.
9. This noun lends itself to the name of a sociological framework that sees society as the product of everyday interactions of individuals. In computing, it also lends itself to the name of a link which is actually a file that points to another file or directory by specifying the latter's path. Vexillological types of these describe information on a flag's recognition, status, and design, while "ticker" types of these are abbreviations used to identify stocks.
12. One ancient philosophy is named after this elusive concept, which is "eternally nameless" and represents the underlying natural order of the universe. The philosophy was founded by *>Sal*, and the concept can also mean "path" or "way". (Although either spelling of this concept in English will be accepted, the Wade–Giles romanisation is intended.)
13. The "eastern" form of these numerals, from zero to nine, are  $\cdot \dot{\vee} \vee \zeta \ominus \bar{\vee} \wedge \wedge$ . Variables in this language can have multiple letters, whereupon they are written cursively to indicate a single variable and not the product of two variables.
14. This theorem was first proven in the 19th century, though in a book still written in Latin. It now has over 240 published proofs. In the book the result is referred to as a fundamental theorem, but the author privately referred to it as the "golden theorem."
15. Albert Einstein derived his theory of special relativity from two of these. The first states that the laws of physics take the same form in all inertial frames of reference, and the second states that the speed of light in free space has the same value  $c$  in all inertial frames of reference.
16. In board sports such as skateboarding, this stance, as opposed to a goofy stance, indicates that a rider has their left foot leading on the board with the right foot pushing. Positive integers of the form  $2^i \cdot 3^j \cdot 5^k$  are known by this adjective. Together with 15 Down (which you may not fill in), it names a set of mathematical objects defined by Ernst Kummer in his work on Fermat's last theorem.
18. In contract bridge, the diamonds and clubs are this kind of suit. In matroid theory, this word describes a matroid  $N$  that is obtained from another matroid  $M$  by a sequence of restriction and contraction operations. This word combines with "Asia" to give another name for Anatolia.
19. This is the last two digits of the year which was designated as the International Year of Peace by the UN, the year in which Halley's comet last came into perihelion. This is the number represented by  $\uparrow$  in cistercian numerals, and the 18th number in the Padovan sequence.

# QUIZDLE — DOWN (HARD)

1. This is the name of a dish consisting of chopped meat, potatoes, and fried onions. Its etymology comes from the French word for “to chop.” In computing, it appears at the start of a shebang.
2. This is the last two digits of the years in which the Prague Spring and Meiji restoration occurred. It is the fifteenth Perrin number and it is the only number with an aliquot sum of 58.
3. In modern portfolio theory, the efficient frontier takes on this shape if no risk-free asset is available. The Shukhov Tower in Polibino was the first of a type of structure described by a derivative of this shape. Antoni Gaudí used the same derivative shape in the Sagrada Família to capture sunlight into the interior of the basilica.
6. In this discipline, several bases – such as blintz, kite, fish, waterbomb, and bird – serve as a starting point after which techniques like the outside crimp and the closed sink can be applied. The Huzita–Hatori axioms and the big-little-big lemma concern this discipline.
7. The sympathetic type of this phenomenon can be seen when bringing a struck tuning fork close to another identical but unstruck tuning fork. In chemistry, this phenomenon, also known as mesomerism, describes bonding in certain molecules by the combination of different contributing structures into a hybrid.
8. This German mathematician and theoretical astronomer studied under Gauss while the latter was directing the Göttingen Observatory. They are recognised for introducing the barycentric coordinate system. In a plane named after this mathematician, every pair of distinct lines intersect in one or two points.
10. This is the title of the studio album by Nas released between King’s Disease II and King’s Disease III, and the studio album by Bruce Springsteen to contain the songs “Radio Nowhere” and “Girls in Their Summer Clothes.” In physics, the numbers 2, 8, 20, 28, 50, 82, and 126 may be described by this adjective.
11. A theorem named for Lasker and this mathematician states that every ideal has a primary decomposition. In 1915, they were invited to become a PD at the University of Göttingen by David Hilbert and Felix Klein, but this was initially blocked by the philosophical faculty.
15. Andrica’s conjecture, Legendre’s conjecture, Chen’s theorem, and Landau’s problems all concern these mathematical objects. The Klauber triangle and Sacks spiral depict these objects. Together with 16 Across (which you may not fill in), it names a set of objects defined by Ernst Kummer in his work on Fermat’s last theorem.
17. The objects studied in nephology are grouped into ten of these. This concept can be applied to knots, graphs, and schemes, and it comes in a “non-orientable” variety.

# QUIZDLE — ACROSS (MEDIUM)

4. This mathematician was a strong proponent of rigour in mathematics, reforming the way that mathematics was taught and done in his country. He was elected a Fellow of the Royal Society in 1910, and wrote the famous essay *3 ? SfZW Sf[US` e 3ba`aYk* in 1940. He is depicted in the photograph below:



5. There are fifteen types of pentagons known to do this in two-dimensional space. This word is also a noun, where it denotes a kind of mathematical object. The Einstein problem asks whether one of these objects with a certain property exists.
9. A type of this named after Schläfli is a notation of the form  $\{p, q, r, \dots\}$  that defines regular polytopes. Types of this named after Legendre, Jacobi, Artin, and Galois have been influential in number theory.
12. A mathematician with this last name resolved the Erdős discrepancy problem in 2015, and made progress on the Collatz conjecture in 2019, where they proved that that almost all Collatz orbits attain almost bounded values. They also coauthored a paper which proved that the sequence of prime numbers contains arbitrarily long arithmetic progressions.
13. Mathematics flourished in this language in the 9th and 10th centuries. By the tenth century, mathematicians working in this language had developed a theory of the multiplication of negative numbers, and discovered the law of sines. A 12th-century Latin translation of *3dfZ WU* written in this language, introduced the decimal positional number system to the Western world.
14. Although probably known to Fermat and Euler, it remained open until Gauss proved it in his landmark 1801 work *6/ecg/ef/a` W 3dfZ WUSW*. Generalising this law to higher powers since then has been a leading problem in mathematics, culminating in class field theory and eventually the Langlands program.
15. This word is derived from the Latin word for “demand”, and has cognates in almost every major European language except Greek and Welsh, where it is instead written  $\alpha\xi\omega\mu\alpha$  or *rhagdybio* respectively.
16. This adjective is used before the word “expression” to denote a sequence of characters that specifies a match pattern in text. In Zermelo–Fraenkel set theory, this adjective gives its name to the axiom also known as the axiom of foundation, which states:
- $$\forall x (x \neq ? \rightarrow \exists y (y \in x \wedge y \cap x = ?)).$$
18. This word describes a graph  $H$  that has been obtained from another graph  $G$  after deleting edges, deleting vertices, and contracting edges. In music, it is another name for the Aeolian mode, and comes in natural, melodic, and harmonic varieties.
19. This is the atomic number of Radon. It is the last two digits of the year in which the Chernobyl disaster occurred. In American English slang, this number is used to indicate that an item is no longer available, traditionally in a restaurant.

# QUIZDLE — DOWN (MEDIUM)

1. This is a shortened name of an oleoresin made by compressing and processing parts of the cannabis plant, which can then be smoked through a pipe, bong, or vaporiser. A mathematical function bearing this name is used to map data of arbitrary size to fixed-size values, typically for use in a table also bearing this name.
2. This is the last two digits of the year in which both Martin Luther King Jr. and Robert F. Kennedy were assassinated. This is the number represented by ⠼⠼ in braille, and the atomic number of erbium.
3. In the Rutherford experiment, if short-range nuclear interactions are ignored, the alpha particle would trace out this shape, as the repulsive Coulomb force satisfies the inverse square law requirement for the Kepler problem. The shadow of a gnomon traces out this shape on any given day.
6. One model in this discipline named after Miura was used to deploy solar panels on spacecraft, and principles from this discipline have been used in the design of stents. Many geometric problems that are unsolvable in Euclidean geometry, such as trisecting the angle and doubling the cube, are solvable in this discipline.
7. In celestial mechanics, the orbital type of this phenomenon occurs when the periods of orbital bodies are related by ratios of small integers, as exemplified by the moons of Jupiter. A dimensionless parameter called the Q factor affects the timespan of this phenomenon; if the Q factor is less than half, this phenomenon will not occur at all.
8. The aforementioned plane is the natural setting for circle inversions, and is also known as the inversive plane. A function named after this mathematician and denoted  $\mu$  can be computed as the sum of the  $n$ -th primitive roots of unity. This function is used in their namesake inversion formula.
10. In programming, this word sometimes describes numbers that have been directly used in source code. In psychology, this word names a kind of thinking in which people believe that unrelated events are causally connected despite the absence of any plausible causal link between them
11. This Jewish mathematician's work has been divided into three epochs. During their second epoch, they investigated rings where certain ascending or descending sequences of ideals must have finite length. Their name now describes general mathematical objects where subobjects this property.
15. These objects can be generated using the sieve of Eratosthenes. Euclid's theorem concerns these objects, and they can be tested for using the AKS or Miller–Rabin testing algorithms. They can be described as safe, illegal, lucky, strong, and industrial-grade.
17. In binomial nomenclature, as introduced by Carl Linnaeus, the first part of the name indicates this attribute, and is always capitalised. The Riemann–Roch theorem relates the complex analysis of a connected compact Riemann surface with this property of the surface.

# QUIZDLE — ACROSS (EASY)

4. In an interview by Paul Erdős, when this British mathematician was asked what his greatest contribution to mathematics was, he unhesitatingly replied that it was the discovery of Ramanujan.
5. The “hat” and “spectre”, discovered in 2022 and 2023, and are aperiodic and mono- types of this, and hence they are solutions to the Einstein problem. The gf/SfW UZMeTaScV bchTW treats dominoes as this kind of object.
9. This word sounds similar to the name given to the round metal plates used as percussion instruments. It also sounds similar to an adjective having the same meaning as “trivial”.
12. This mathematician was the youngest IMO participant to win a bronze, silver, or gold medal. They are popularly known as “the Mozart of Maths.”
13. The English word “algebra” can be traced back to the usage of SŽSTd in this language by Persian mathematician al-Khwarizmi. Today, it is an official language in many countries including Libya and Yemen.

14. This law states that

$$\frac{p}{q} \cdot \frac{q}{p} = (-1)^{\frac{p-1}{2} \frac{q-1}{2}}.$$

where

$$\frac{q}{p} = \begin{cases} 1 & \text{if } n^2 \equiv q \pmod{p} \text{ for some integer } n, \\ -1 & \text{otherwise.} \end{cases}$$

15. One of these named after Bertrand states that for every  $n > 1$ , there is always at least one prime  $p$  such that  $n < p < 2n$ . Euclid stated five of these at the start of his 7W Wfē, the last of which is known as the “parallel” one.
16. This adjective describes a graph whose vertices all have equal degree. The five Platonic solids can also be described as convex polyhedra with this additional property. In sports league, this name is given to one of the four seasons, the other three being preseason, postseason, and off-season.
18. In linear algebra, this is the name given to the determinant of a square submatrix. This word is also used in law to describe someone who has not yet reached the age of majority.
19. This number is the calling code of China, and twice the calling code of Austria. When this number is preceded with “x”, the result denotes a family of CISC instruction set architectures which are used in most computers today.

