

# FUNCTIONS — SOLUTIONS

- Teams may try the functions in any order, and may come back to functions later.
- Where a 2D table is used, the first column represents the  $x$ -values and the first row represents the  $y$ -values.

F1.  $f(x, y) = \binom{x}{y}$ .

$x \setminus y$	2	3	4	5	6	7	8	9	10
2	1	0	0	0	0	0	0	0	0
3	3	1	0	0	0	0	0	0	0
4	6	4	1	0	0	0	0	0	0
5	10	10	5	1	0	0	0	0	0
6	15	20	15	6	1	0	0	0	0
7	21	35	35	21	7	1	0	0	0
8	28	56	70	56	28	8	1	0	0
9	36	84	126	126	84	36	9	1	0
10	45	120	210	252	210	120	45	10	1

F2.  $f(x, y) = x^2 - xy + y^2$ .

$x \setminus y$	2	3	4	5	6	7	8	9	10
2	4	7	12	19	28	39	52	67	84
3	7	9	13	19	27	37	49	63	79
4	12	13	16	21	28	37	48	61	76
5	19	19	21	25	31	39	49	61	75
6	28	27	28	31	36	43	52	63	76
7	39	37	37	39	43	49	57	67	79
8	52	49	48	49	52	57	64	73	84
9	67	63	61	61	63	67	73	81	91
10	84	79	76	75	76	79	84	91	100

F3.  $f(x, y) = 2^x + y^3$ .

$x \setminus y$	2	3	4	5	6	7	8	9	10
2	12	31	68	129	220	347	516	733	1004
3	16	35	72	133	224	351	520	737	1008
4	24	43	80	141	232	359	528	745	1016
5	40	59	96	157	248	375	544	761	1032
6	72	91	128	189	280	407	576	793	1064
7	136	155	192	253	344	471	640	857	1128
8	264	283	320	381	472	599	768	985	1256
9	520	539	576	637	728	855	1024	1241	1512
10	1032	1051	1088	1149	1240	1367	1536	1753	2024

F4.  $f(x, y) = \frac{x}{y} + \frac{y}{x}$ .

$x \backslash y$	2	3	4	5	6	7	8	9	10
2	2	2.167	2.5	2.9	3.333	3.786	4.25	4.722	5.2
3	2.167	2	2.083	2.267	2.5	2.762	3.042	3.333	3.633
4	2.5	2.083	2	2.05	2.167	2.321	2.5	2.694	2.9
5	2.9	2.267	2.05	2	2.033	2.114	2.225	2.356	2.5
6	3.333	2.5	2.167	2.033	2	2.024	2.083	2.167	2.267
7	3.786	2.762	2.321	2.114	2.024	2	2.018	2.063	2.129
8	4.25	3.042	2.5	2.225	2.083	2.018	2	2.014	2.05
9	4.722	3.333	2.694	2.356	2.167	2.063	2.014	2	2.011
10	5.2	3.633	2.9	2.5	2.267	2.129	2.05	2.011	2

F5.  $f(n) = 2024 \pmod{n}$ .

Read table by (first digit of  $n$ ) \ (last digit of  $n$ ).

first \ last	0	1	2	3	4	5	6	7	8	9
0	-	0	0	2	0	4	2	1	0	8
1	4	0	8	9	8	14	8	1	8	10
2	4	8	0	0	8	24	22	26	8	23
3	14	9	8	11	18	29	8	26	10	35
4	24	15	8	3	0	44	0	3	8	15
5	24	35	48	10	26	44	8	29	52	18
6	44	11	40	8	40	9	44	14	52	23
7	64	36	8	53	26	74	48	22	74	49
8	24	80	56	32	8	69	46	23	0	66
9	44	22	0	71	50	29	8	84	64	44

$n = 100$ : answer = 24.

# SHUTTLE — SOLUTIONS

- If an answer is incorrect, make sure the other pair is aware.
- Circle the corresponding number: 4 if correct on the first attempt, 3 if correct on the second attempt, and 0 otherwise.
- Record the time that the time finished. If the team finished in faster than 8 minutes, circle 2, if faster than 10 minutes circle 1, otherwise circle 0.
- Fill in the total number of points achieved.

A1. 25

A2. 405

A3. 305

A4. 22

B1. 1840

B2. 84

B3. 49

B4. 3

# RELAY — SOLUTIONS

Team:

Referee:

(Circle the corresponding numbers and add them up at the end.)

**R1**

2022

4 3 0

**R2**

$3/(2\sqrt{7})$

4 3 0

**R3**

2

4 3 0

**R4**  $101 \cdot 302/12 = 15251/6$

4 3 0

**R5**

97

4 3 0

**R6**  $2^{1012} + 1012 \cdot 2^{506} - 1$

4 3 0

Time

4 2 0

/28

# CROSSNUMBER — SOLUTIONS

- The pairs are supposed to only attempt one square at a time. If many squares are filled in at once, stop marking at the first incorrect square and consult the pair.
- If the answer for a square is correct, tick the circle in the lower right.
- If the answer for a square is incorrect, cross it out and write the correct answer.
- The pairs are responsible for copying the correct answers onto their own grids.
- The two pairs are not allowed to communicate on any matters related to the questions themselves, but may encourage the other team to, for example, work on a specific clue.
- Teams are not allowed to communicate what values they think are possible for a square.

<sup>1</sup> 7			<sup>2</sup> 6	7	<sup>3</sup> 6	5
<sup>4</sup> 1	0	<sup>5</sup> 2	0		7	
4		4			<sup>6</sup> 2	7
		<sup>7</sup> 3	3	<sup>8</sup> 3	3	
<sup>9</sup> 8	<sup>10</sup> 4	1		1		
	6			<sup>11</sup> 1	1	<sup>12</sup> 3
	<sup>13</sup> 8	0	6	4		0

# QUIZDLE — SOLUTIONS

- Allow and fix minor spelling mistakes.
- If the answer is incorrect, don't give the correct answer, and the team is not allowed to attempt that word again, even using easier clues.
- Let your team fill in the crossword – your job is just to mark the Answer Sheet.
- The Across and Down clues of a particular difficulty must be given out at the same time.

## Across

4. HARDY
5. TILE
9. SYMBOL
12. TAO (accept DAO but fix spelling)
13. ARABIC
14. QUADRATICRECIPROCITY
15. POSTULATE
16. REGULAR
18. MINOR
19. EIGHTYSIX

## Down

1. HASH
2. SIXTYEIGHT
3. HYPERBOLA
6. ORIGAMI
7. RESONANCE
8. MOBIUS
10. MAGIC
11. NOETHER
15. PRIME
17. GENUS