



ICMC 9 — Round Two

22 February 2026

Name: _____

Contestant ID: _____

Institution/University: _____

Instructions:

- You will have 4 hours to solve 5 problems, each of which carries 10 marks.
- Use a black or blue pen or a dark pencil. Rulers, compasses, protractors, and erasers may be used but will not be required. All electronic devices, including calculators, are prohibited.
- Drinks are allowed, but food is prohibited.
- Write your solution to each problem on a different page. At the top of each page, write down the question number, your initials, and your contestant number. Use both sides whenever possible. Write clearly and not too faintly – your work will be scanned for marking.
- Problems are listed roughly in order of difficulty. Proofs are expected for all problems even if they only ask for an answer.
- One complete solution will be awarded more marks than several unfinished attempts.
- You may not leave the contest venue in the first two hours or the last thirty minutes unless exceptional circumstances arise.
- This cover page must be completed and placed facing up on top of the scripts you wish to submit. You may take any rough work with you.

DO NOT TURN OVER UNTIL INSTRUCTED

TICK ATTEMPTED PROBLEMS					
Problem	1	2	3	4	5
Attempt?					

Problem 1. Prove that for all odd-degree polynomials p , there exist real numbers a and c and a non-zero real number d such that $f(a - d) + c = f(a) = f(a + d) - c$.

Problem 2. On the Euclidean plane, there are 11 bells of distinct tones and 2026 stationary villagers. All 11 bells are simultaneously rung exactly once. Supposing that sound travels at a constant speed, and that no villager hears two or more bells at the same time, prove that there are two villagers who hear the same order of bell tones regardless of where the bells and the villagers are.

Problem 3. Let a and p be positive integers with p prime, and let $n = p^a$. Define $f : \mathbb{Z}^n \rightarrow \mathbb{Z}^n$ by

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{pmatrix} \mapsto \begin{pmatrix} x_1 \\ x_1 + x_2 \\ x_1 + x_2 + x_3 \\ \vdots \\ x_1 + x_2 + x_3 + \cdots + x_n \end{pmatrix}.$$

Find, in terms of a and p , the smallest value of k such that, for every $x \in \mathbb{Z}^n$, n divides every component of $f^k(x) - x$. (Here, f^k denotes k iterated applications of f .)

Problem 4. Let P be a point strictly inside an equilateral triangle ABC of area 1. Let ℓ_A be the result of rotating the line AP by 60° clockwise about A . Define ℓ_B and ℓ_C similarly. What is the minimal possible area of the triangle formed by ℓ_A , ℓ_B , and ℓ_C ?

Problem 5. What is the largest positive integer n such that there exists a complex number $a \in \mathbb{C}$ for which the numbers a, a^2, \dots, a^n are all distinct and lie on a single non-degenerate parabola in the complex plane?

A parabola in the complex plane is a set of the form

$$\{x + yi \in \mathbb{C} : ax^2 + bxy + cy^2 + dx + ey + f = 0\},$$

where a, b, c, d, e, f, x , and y are real numbers; and a, b , and c satisfy the equation $b^2 - 4ac = 0$ and are not all 0. The parabola is non-degenerate if and only if the polynomial $ax^2 + bxy + cy^2 + dx + ey + f$ does not factor into two polynomials of degree one. This description of a non-degenerate parabola corresponds to all rotations and dilations of the standard parabola $y = x^2$ in the Cartesian plane.