# Functions Answer sheet

Team:

Referee:

## F1

Consider the function $f_1$ from $\{1, \ldots, 10\} \times \{1, \ldots, 10\}$ to the positive integers.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Inputs 1: | | | | | | | | | | |
| Inputs 2: | | | | | | | | | | |
| Outputs: | | | | | | | | | | |

Description:

## F2

Consider the function $f_2$ from $\{1, \ldots, 10\} \times \{1, \ldots, 10\}$ to the integers.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Inputs 1: | | | | | | | | | | |
| Inputs 2: | | | | | | | | | | |
| Outputs: | | | | | | | | | | |

Description:

# F3

Consider the function $f_3$ from $\{1, \ldots, 10\}$ to the positive integers.

Inputs:

Outputs:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | | | | | | | | | |

Description:

# F4

Consider the function $f_4$ from $\{1, \ldots, 10\}$ to the positive integers.

Inputs:

Outputs:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | | | | | | | | | |

Description:

# F5

Consider the function $f_5$ from $\{1, \ldots, 10\}$ to the positive integers.

Inputs:

Outputs:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | | | | | | | | | |

Description:

# Shuttle — A1 and A3

## A1

If we choose three distinct vertices of a cube uniformly at random, and the probability of them forming a right-angled triangle is $\frac{p}{q}$ in its reduced form (so $p, q \in \mathbb{N}$, $\gcd(p, q) = 1$), determine the value of $q$.

Pass on your answer to A1 as $X$.

## A3

$Y$ is the number you will receive.

In the centre of the bottom-left corner square of a $Y \times Y$ board there is a frog. It can freely jump around the board as long as every jump it makes does not land below the row it jumps from, or to the left of the column it jumps from. It must also move to a new square in every jump. In how many ways can the frog land in the upper-right corner of the board after exactly 3 jumps?

Pass on your answer to A3 as $Z$.

# Shuttle — A2 and A4

## A2

$X$ is the number you will receive.

A certain number of bishops is placed on the $X \times X$ chess board such that every single one of them attacks an even number of other bishops. Determine the minimum number of empty spaces that can be left on the chess board.

Pass on your answer to A2 as $Y$.

## A4

$Z$ is the number you will receive.

Two players play the following game: at the beginning, a number $n \in \mathbb{N}$ is written on the board. Then, starting from the first player, they alternately erase the number currently on the board and replace it with the difference of that number and one of its divisors of their choice, not equal to 1 or the number itself. The player who cannot make a move loses.

Determine for how many values of $n \in \{1, 2, \ldots, 10 \cdot Z\}$ the first player has a winning strategy.

# Shuttle — B1 and B3

## B1

Let $x$ be a natural number with 101 digits with eleven 1's, and with every two consecutive 1's separated by nine 0's (so $x$ has no other digits). Determine the sum of digits of number $x^2$.

Pass on your answer to B1 as $X$.

## B3

$Y$ is the number you will receive.

Suppose points $A, B, C$ and $D$ lie on a circle in that order clockwise, and let $P$ be the point on line segment $AD$ such that $BP$ and $CD$ are parallel to each other. If $|PA| = Y$, $|PB| = 2 \cdot Y$ and $|PC| = 4 \cdot Y$, find the length $|AD|$.

Pass on your answer to B3 as $Z$.

# Shuttle — B2 and B4

## B2

$X$ is the number you will receive.

Calculate in how many ways we can tile a $2 \times \lfloor \sqrt{X} \rfloor$ rectangular board using $1 \times 2$ rectangles, $2 \times 1$ rectangles, and $2 \times 2$ squares. (Reflections and rotations count as distinct tilings.)

Pass on your answer to B2 as $Y$.

## B4

$Z$ is the number you will receive.

The vertices of a regular $Z$-gon are coloured in two colours such that there is no monochromatic regular $k$-gon for $k \leq Z$ with vertices among these points. Find the minimum number of vertices that can be coloured using the less frequent colour.

# Shuttle — Answer sheet A

Team:

Referee:

| | | | | | |
|---|---|---|---|---|---|
| **A1** | | | | 4 | 3 0 |
| **A2** | | | | 4 | 3 0 |
| **A3** | | | | 4 | 3 0 |
| **A4** | | | | 4 | 3 0 |

Time: 2 1 0

Final Score: / 18

# Shuttle — Answer sheet B

Team:

Referee:

| | | | | | |
|---|---|---|---|---|---|
| **B1** | | | | 4 | 3 0 |
| **B2** | | | | 4 | 3 0 |
| **B3** | | | | 4 | 3 0 |
| **B4** | | | | 4 | 3 0 |

Time: 2 1 0

Final Score: / 18

# Relay — R1

Team: _____

The first three moves of a chess game (that is, two moves by the first player, and one by the second player) are played with knights only. In how many different ways can this be done?

First attempt

Second attempt

# Relay — R2

Team: _____

Teeny, Tiny, and Tony, set off on a 100-km journey. Each walks at 5 kph, and they have a car which always drives at 65 kph, but it can only fit two people. First, Teeny walks while Tiny and Tony drive. At some point, Tiny drops Tony off to walk to the end, while Tiny goes back and picks up Teeny before driving to the end. It turns out that all three people finished the journey at the same time. How far did the car drive in total?

First attempt

Second attempt

# Relay — R3

Team: _____

Let $A$, $B$ and $C$ be three distinct midpoints of a cube's edges. Over all possible choices of $A$, $B$ and $C$, what is the largest possible value of $\angle ABC$ (in degrees)?

First attempt

Second attempt

# Relay — R4

Team: _____

In how many ways can we colour a $3 \times 3$ grid with 4 colours such that no row or column contains two cells coloured with the same colour?

First attempt

Second attempt

# Relay — R5

Team: _____

A natural number $n$ is called good if $n$ divides $\lfloor (n-1)\sqrt{n} \rfloor$. Find the number of good natural numbers not greater than 2026.

First attempt

Second attempt

# Relay — R6

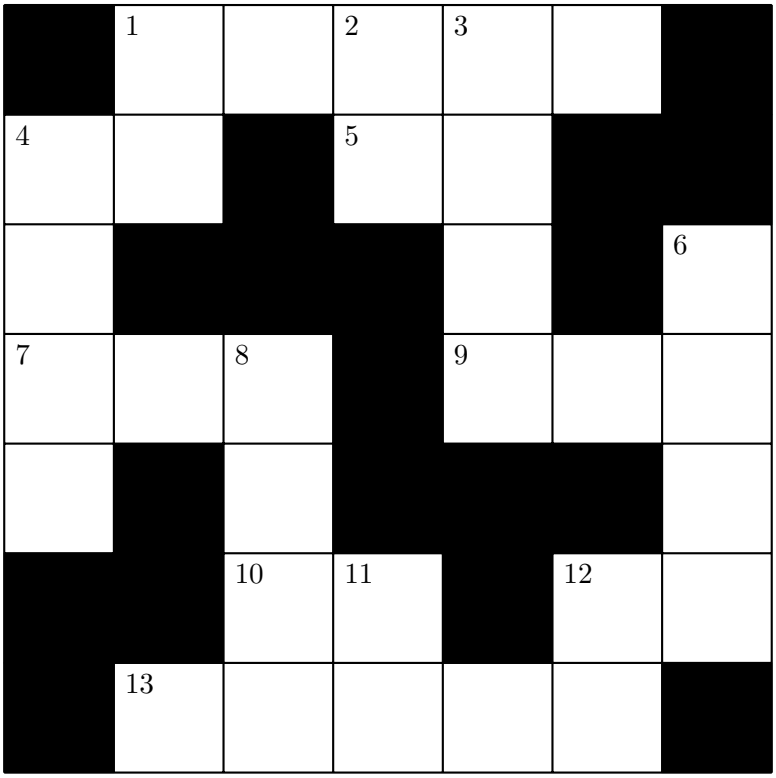
Team: _____

Let $f(x) = ax^2 + bx + c$ for some $a, b, c \in \mathbb{R}$. If $|f(0)| \leq 1$, $|f(-1)| \leq 1$ and $|f(1)| \leq 1$, what is the largest possible value that $\max_{x \in [-1,1]} |4f(x)|$ can have?

First attempt

Second attempt

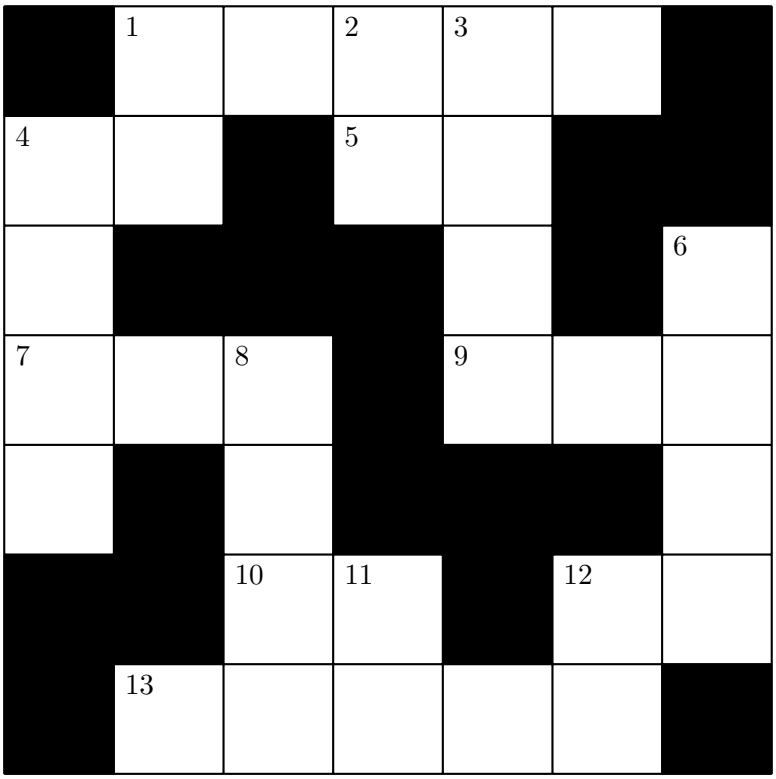
# Crossnumber — Across




## Across

1. $x^4$, where $x$ is the sum of the digits in the third column.
4. A number of the form $2n^2$, where $n$ is an integer.
5. A prime.
7. 360 + sum of digits of (4 Across).
9. The sum of its digits is (12 down).
10. Less than $\sqrt[3]{(6\text{ Down})}$.
12. Same as (1 Down).
13. A palindrome.

# Crossnumber — Down



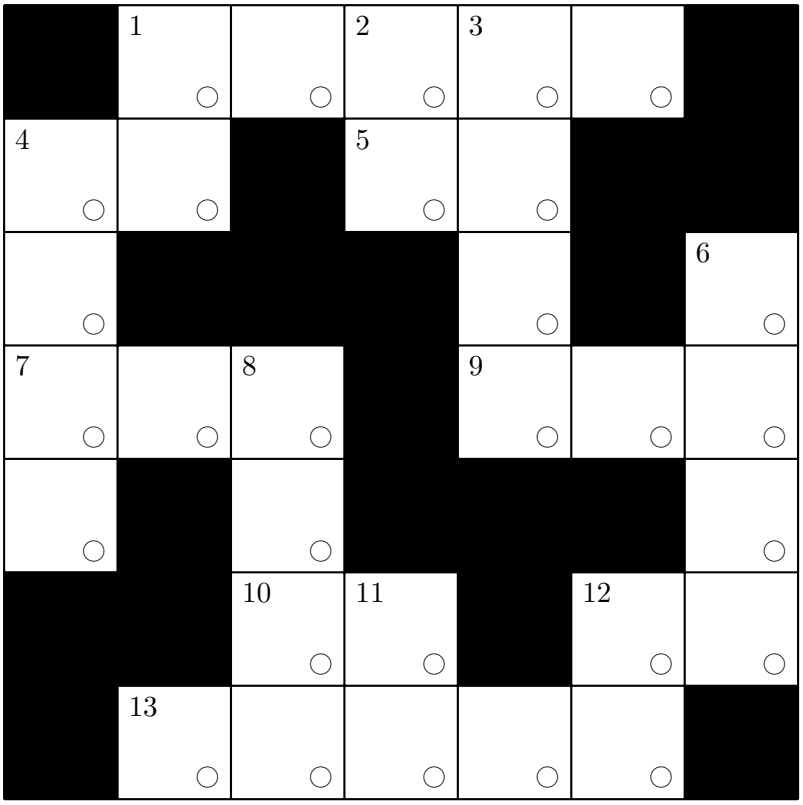

## Down

1. This number is the number of occurrences of $x$ in the completed crossnumber, where $x$ is the most common number in the completed crossnumber.
2. A multiple of the second digit of (4 Down).
3. A palindrome.
4. The digits of this number are distinct and odd.
6. Two times the product of the digits of (4 Down).
8. This number is odd and its first three digits are distinct.
11. Three times the last digit of (4 Down).
12. A multiple of 12.

# Crossnumber — Answer sheet

Team:

Referee:




Totals

/5

/4

/3

/6

/3

/4

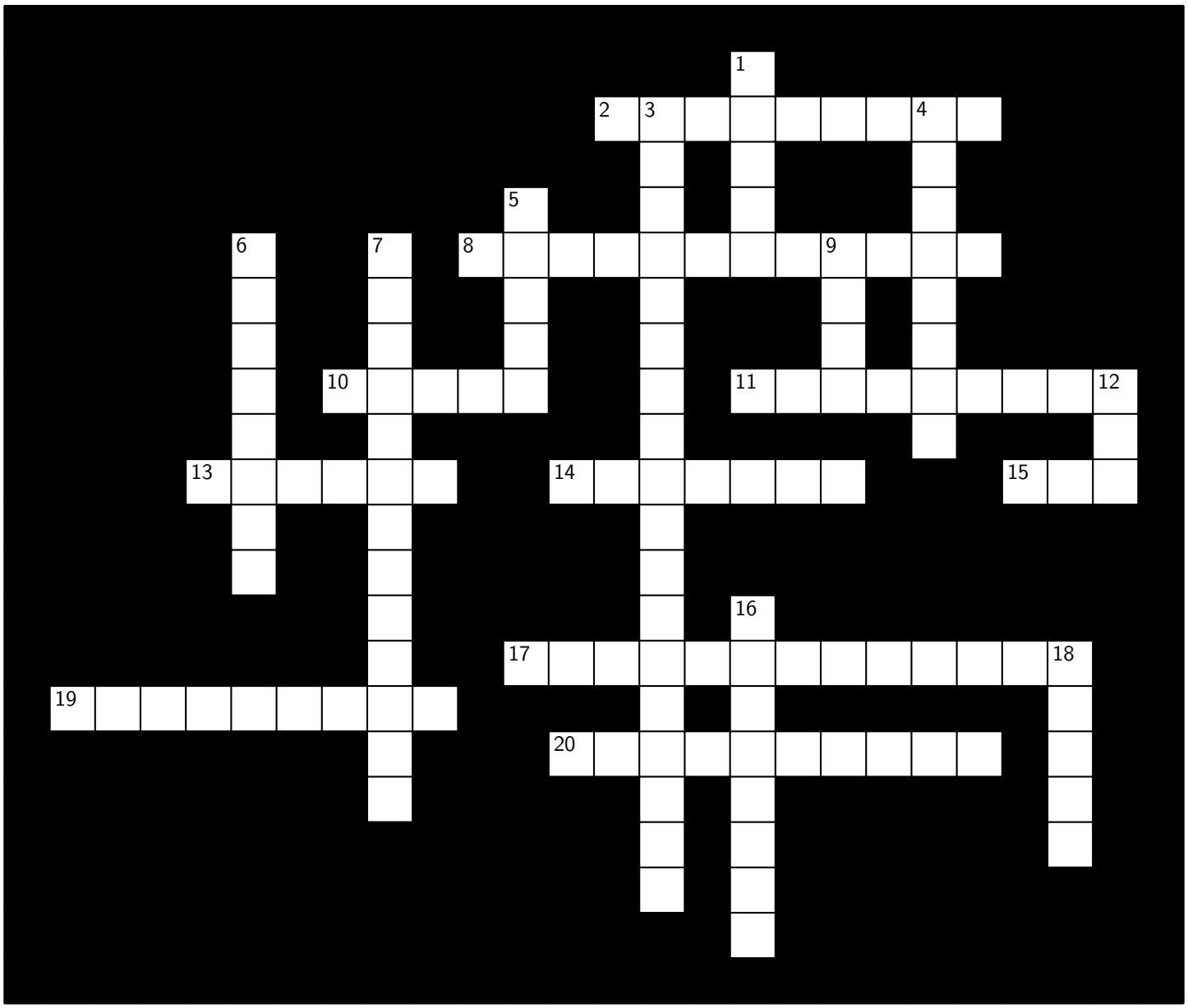
/5

/30

# Quizdle — Crossword Sheet

Team:

Referee:



# Quizdle — Answer Sheet

Team: _____

Referee: _____

| | **Across** | H | M | E |
|---|---|---|---|---|
| **2** | | $1\frac{1}{2}$ | 1 | $\frac{1}{2}$ |
| **8** | | $1\frac{1}{2}$ | 1 | $\frac{1}{2}$ |
| **10** | | $1\frac{1}{2}$ | 1 | $\frac{1}{2}$ |
| **11** | | $1\frac{1}{2}$ | 1 | $\frac{1}{2}$ |
| **13** | | $1\frac{1}{2}$ | 1 | $\frac{1}{2}$ |
| **14** | | $1\frac{1}{2}$ | 1 | $\frac{1}{2}$ |
| **15** | | $1\frac{1}{2}$ | 1 | $\frac{1}{2}$ |
| **17** | | $1\frac{1}{2}$ | 1 | $\frac{1}{2}$ |
| **19** | | $1\frac{1}{2}$ | 1 | $\frac{1}{2}$ |
| **20** | | $1\frac{1}{2}$ | 1 | $\frac{1}{2}$ |

| | **Down** | H | M | E |
|---|---|---|---|---|
| **1** | | $1\frac{1}{2}$ | 1 | $\frac{1}{2}$ |
| **3** | | $1\frac{1}{2}$ | 1 | $\frac{1}{2}$ |
| **4** | | $1\frac{1}{2}$ | 1 | $\frac{1}{2}$ |
| **5** | | $1\frac{1}{2}$ | 1 | $\frac{1}{2}$ |
| **6** | | $1\frac{1}{2}$ | 1 | $\frac{1}{2}$ |
| **7** | | $1\frac{1}{2}$ | 1 | $\frac{1}{2}$ |
| **9** | | $1\frac{1}{2}$ | 1 | $\frac{1}{2}$ |
| **12** | | $1\frac{1}{2}$ | 1 | $\frac{1}{2}$ |
| **16** | | $1\frac{1}{2}$ | 1 | $\frac{1}{2}$ |
| **18** | | $1\frac{1}{2}$ | 1 | $\frac{1}{2}$ |

Minor spelling mistakes are allowed and will be fixed by the marker.

Round down sum of scores to the nearest integer to get...

Final Score: _____ / 30

# Quizdle — Across (Hard)

2. In mathematics, a homotopy theory described by this adjective is a subfield of stable homotopy theory that studies complex-oriented cohomology theories. The Hadwiger–Nelson problem studies a number described by this adjective. The quantum theory explaining the strong nuclear force uses the root of this adjective in its name, as it describes how quarks and gluons interact with other particles.

8. One three-dimensional solution to this problem with no singularities corresponds to a Hopf fibration. In two dimensions, smooth solutions to this problem exist globally. A statement of the conservation of mass is generally required in applications of this, and this is achieved by adding mass continuity.

10. A technique to solve homogenous inequalities in three variables by representing their coefficients in a triangular array is named after this country. Mathematics formed one of this country's Six Arts, which also included Music and Archery. A mathematician from this country generalised the Gauss–Bonnet theorem to higher manifolds by developing their theory of fibre bundles.

11. This is the name of the board game which won the 1994 Spiel des Jahres award, in which players aim to construct and control skyscrapers. In LASSO regression analysis, a measure bearing this name is used as the penalty.

13. In economics, a non-linear financial model can be described by this term. This word describes metric spaces where for any two distinct points $X$ and $Y$, there is a point $Z$ distinct from $X$ and $Y$, such that $d(X,Z) + d(Z,Y) = d(X,Y)$. In combinatorial geometry, the Hadwiger conjecture states that any body in $n$-dimensional space with this property can be covered by $2^n$ or fewer smaller bodies similar to the original body.

14. In algebra, this kind of group and this kind of ring exist, but this kind of field does not. The word derives from the name for the first three of the seven liberal arts, comprising grammar, logic, and rhetoric. The tangent bundle of a Lie group has this property.

15. This word unites all of the following. C. Northcote Parkinson said "work expands so as to fill the time available for its completion". Charles Goodhart said "When a measure becomes a target, it ceases to be a good measure". Frank Benford said that in many real-life sets of numerical data, the leading digit is likely to be small, with 1 being the leading digit about 30% of the time.

17. It is an open problem as to whether the partition principle implies this statement. This statement is used to prove the existence of Vitali sets, and that every field has an algebraic closure. In type theory, this statement can be expressed as $(\forall x^{\sigma})(\exists y^{\tau})R(x,y) \to (\exists f^{\sigma\to\tau})(\forall x^{\sigma})R(x,f(x))$.

19. This mathematician failed their university exams due to an excessive focus on mathematics over their other courses and ran away from home.

20. This is the only solution for $N$ greater than 1 in the diophantine equation $1^2 + 2^2 + \cdots + N^2 = M^2$. This is the number of Tirthankaras in Jainism. In Christianity, it represents the complete church, being the sum of the number of tribes of Israel and the number of Apostles of the Lamb of God. In Brazil, this number is associated with homosexuality as it is the number that stands for the deer in a game known as "jogo do bicho".

# Quizdle — Down (Hard)

1. The *Tonnetz*, a lattice representing tonal space in western music theory, is isomorphic to a flat one of these. The trefoil knot is a $(2, 3)$ one of these kinds of knots. In medicine, this kind of fracture, also known as a buckle fracture, is the most common type of bone fracture in children.

3. In this field, one may encounter apeirogons in horocycles, as well as the Beltrami–Klein model. An alternative name for this field popular in the former Soviet Union is one named after Lobachevsky. Althoguh Gauss explored this field extensively, he did not publish anything about it due to fears that it would tarnish his reputation as "the Prince of Mathematicians".

4. When stylised in capital letters, this gives the name of a European space agency telescope active from 2002 to 2025 used to observe gamma rays with energies of up to 8 MeV. As a symbol, it is very similar to the symbol used in phonology to represent the "sh" sound in English. In commutative algebra, an element $b$ of a commutative ring $B$ is said to be this over a subring $A$ of $B$ if $b$ is a root of some monic polynomial over $A$.

5. This is the name of a computer algebra system produced by the University of Sydney Computational Algebra Group which is used extensively within pure mathematics. It is also the sixth studio album by French heavy metal band Gojira.

6. In aerodynamics, the flight one of these, represented by a doghouse plot, refers to the capabilities of an aircraft in terms of airspeed and load factor or altitude. In category theory, this names a construction that generalises the operations of "exterior completion", such as the Stone–Čech compactification of a topological space. It is the dual construction of refinement.

7. This mathematician worked under the pseudonym of Monsieur Le Blanc, corresponding with Lagrange, Legendre, and Gauss on topics including number theory. They were a pioneer of elasticity theory, and won the grand prize from the Paris Academy of Sciences for their work.

9. In logic, this word describes a formula that has at least one free variable, as opposed to formulas in which all variables are bound to a quantifier. In complex analysis, a result states that if $U$ is a domain of the complex numbers and $f : U \to \mathbb{C}$ is a non-constant holomorphic function, then $f$ is one of these maps.

12. This is the adjective attached to a paradigm of mathematics pedagogy used from the 1950s to the 1970s which tried to bring mathematics education closer to the research that was being done by pure mathematics, and which emphasised learning by discovery. A different form of this word describes a kind of logicism viewed as the successor to Frege's original logicist programme after Russell's paradox demonstrated its inconsistencies.

16. This number is represented by MB´ in Greek numerals, and is the atomic number of molybdenum. This number is five times less than the fourth primorial number. It is the only jersey number retired by all Major League Baseball teams.

18. A kind of diagram named after this person is often considered to be a generalised form of a Venn diagram. They popularised the use of $i$ to represent the imaginary unit and the notation $f(x)$ to denote function application. They were devoutly religious throughout their life, and became blind towards the end of their life.

# Quizdle — Across (Medium)

2. In music, a scale described by this adjective uses all 12 notes of the octave. For the plane, the number described by this adjective is known to be 5, 6, or 7, after biologist Aubrey de Grey proved that it was not 4 in 2018. Humans typically have 46 of these, which also shares a root with this adjective.

8. These generalise Euler's equations to the case of nonzero viscosity. The 2017 movie Gifted revolves around a 7-year-old maths prodigy whose mother's work was on this topic.

10. The mathematician named in the hard clue is the namesake of the Chern medal. A mathematician born in this country established the first finite bound on gaps in the prime numbers in their work on the twin prime conjecture. Negative numbers appear for the first time in history in this country's *Nine Chapters on the Mathematical Art*.

11. The geometry bearing this name uses the $L_1$ norm, and in this geometry, "circles" appear as diamonds. It is also the name of a American war-time project associated with the phrase "Now I am become Death, the destroyer of worlds".

13. A hull known by this adjective is a common technique used to solve combinatorial or discrete geometry problems. Minkowski's theorem is the statement that every $n$-dimensional set with this property and symmetric about the origin with volume greater than $2^n$ contains a non-zero integer point. In order to use Jensen's inequality with a function, that function must satisfy this property.

14. In a online meme, a Reddit post lists over two hundred synonyms of this word, which include "Terry Tao told me in a personal email that..." and "Assuming the reader's intellect approaches that of the writer, it should be obvious that...". A group action on a set is called this if every group element acts as the identity on the set.

15. The Code of Hammurabi and the Code of Justinian are two famous texts concerning this, and in the modern day it comes in many varieties, such as civil, tort, and common. One named after Steven Stigler states that no scientific discovery is named after its original discoverer.

17. The Banach–Tarski paradox critically requires this ingredient, and Tarski often used this paradox to show that this led to counterintuitive results. The Schröder–Bernstein theorem, which states that the existence of two injective functions $A \to B$ and $B \to A$ implies the existence of a bijection between them, was proven in 1895 by Cantor assuming this statement, but later other proofs were found which did not assume this statement.

19. This mathematician is known for attributing their discoveries to their family goddess in their vivid dreams. They are known for saying "an equation for me has no meaning unless it is a thought of god".

20. This number is equal to one sixth of a gross, and is the kissing number in four-dimensional space. It is also the number of spokes in the Ashoka Chakra as depicted on the flag of India. This number is one of the regular $n$-cells that exist in four dimensions.

# Quizdle — Down (Medium)

1. This mathematical object is associated with the game *Asteroids*, and it can be written as $\mathbb{R}^2/\mathbb{Z}^2$. Polyhedra which can be described by the adjectival form of this word do not satisfy $V - E + F = 2$.

3. The "opposite" of this field, which concerns objects with negative curvature, is one called "elliptic", which concerns objects with positive curvature. Development of the video game *HyperRogue* relied heavily on the existence of this field.

4. Diophantine equations are concerned with equations to be solved over a domain described by this adjective. The modern notation was first publicly used in an article titled *De Geometria Recondita et analysi indivisibilium atque infinitorum*. The mean value theorem has two variants which concern these.

5. This is the name of the algebraic structure which can be turned into a group by adding the properties of associativity, identity, and invertibility.

6. In music, the amplitude type of this affects the volume of a sound over time, and they are commonly generated using the attack, delay, sustain, and release parameters. In a paradox involving two of these, it appears that choosing to switch between them increases the expected amount of money to five quarters of the original amount. Putting money into these is also a personal budgeting strategy described by this word.

7. For every safe prime, there exists a prime named after this mathematician. As their parents did not approve of them studying mathematics, they studied it secretly at night.

9. A function $f : \mathbb{R} \to \mathbb{R}$ is continuous if and only if the preimage of every set described by this term is a set described by this term. This is the principle which later led to these sets being used to define a topological space. The word appears in the phrase from which the acronym FOSS or FLOSS is derived, in reference to software.

12. The Greek root of this word appears in the name of the dominant approach to modern microeconomics in which the production, consumption, and valuation of goods and services are driven by the demand and supply model. The same root appears in the adjective describing the retention of juvenile physiological traits into adulthood.

16. According to Douglas Adams, this is the answer to the question "What do you get if you multiply six by nine?" This is the 6th pronic or rectangular number, and the 5th Catalan number. It is the magic constant of the smallest $3 \times 3 \times 3$ magic cube.

18. It is said that this mathematician has so many things named after them that some discoveries were named after the second person to discover them, although the inverse probably happened more often. This mathematician names a constant, sometimes along with another mathematician, whose value begins with 0.577.

# Quizdle — Across (Easy)

2. The 24th element of the periodic table has the same root as this adjective, and it is so named because many of its compounds are intensely coloured. Objects that have only one colour are described by prepending the prefix "mono-" to this adjective.

8. For the three-dimensional system of equations, and given some initial conditions, mathematicians have neither proved that smooth solutions always exist, nor found any counter-examples. This open problem is one of the seven Millennium Prize Problems.

10. This country is the most successful country at the International Mathematical Olympiad, after they participated for the first time in 1985. Despite representing Australia at the International Mathematical Olympiad, Terence Tao is of this ethnicity, which plays into the joke that there are actually multiple teams representing this country at the International Mathematical Olympiad.

11. The metric bearing this name is also known as taxicab or rectilinear distance. This is the most densely populated borough of New York city, and also refers to one of its islands. A project named after this island was the subject of the 2023 film *Oppenheimer*.

13. A set in the Euclidean plane can be described by this adjective if it satisfies the following property: the line segment joining any two points in the set lies in the set. The word describes a function whose second derivative is nonnegative everywhere. A lens described by this adjective focuses rays of light.

14. The solutions of the Riemann zeta function that lie outside the critical strip are known by this name. Synonyms include "easy", "simple", and "uninteresting", and use of the word is discouraged in academia.

15. One named after "large numbers" states that the as the number of samples increases, the sample mean converges to the true mean. One named after Snell says that the ratio of the sines of the angle of incidence and the angle of refraction is equal to the ratio of the refractive indices.

17. In 1963, Paul Cohen developed his technique of forcing to show that this statement is logically independent from the Zermalo–Frankel axioms. It states that, given any collection of non-empty sets, it is possible to construct a new set by choosing one element from each set.

19. This mathematician's life is depicted in the 2015 film *The Man Who Knew Infinity*, which demonstrates his relationship with British mathematicians Hardy and Littlewood.

20. In a children's mathematical game, it is difficult, but possible, to make this number using the four basic arithmetic operations and the numbers 5, 5, 5, and 1 exactly once each. It is the number of square faces in a tesseract.

# Quizdle — Down (Easy)

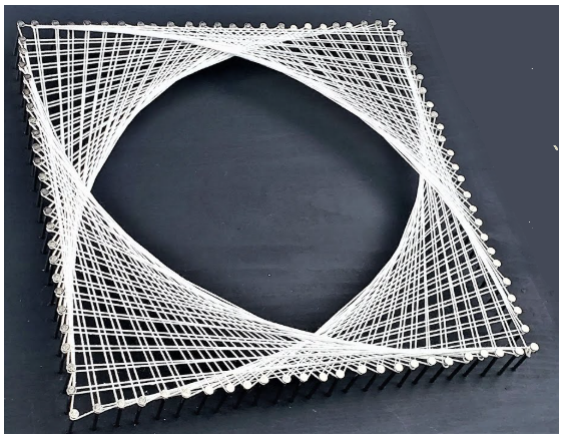
1. This word sounds the same as the name for the astrological sign ♉, which corresponds mostly with the month of May in western astrology. It has the same shape as a bagel, and is topologically equivalent to a straw.

3. In this kind of geometry, lines are often drawn as arcs and are sometimes defined as geodesics. It is the result of forgoing Euclid's parallel postulate.

4. This comes in Lebesgue and Riemann forms, and the ancient Greeks computed these using thin rectangles, trapeziums, and the method of exhaustion.

5. This is the name for the molten rock found under Earth's surface, becoming lava when it breaks the Earth's surface.

6. Chinese red packets are also known as red ones of these. In geometry, some families of planar curves have one of these, which is a curve tangent to each member of the family at some point. It appears frequently in string art, for example in the following image:



7. In her work on Fermat's Last Theorem, she proved in a theorem named after her that if the exponent is an odd prime $p$ and $2p + 1$ is also prime, then $p$ must divide $x$, $y$, or $z$.

9. This word is in the name of the organisation that released ChatGPT. A problem in mathematics is described by this adjective if it is unsolved.

12. This word appears in the name of the English polymath who developed infinitesimal calculus at the same time as Leibniz. The Greek root of this word is the namesake for the 10th element of the periodic table, the second noble gas.

16. This number can be obtained by multiplying the two most memed numbers in 2025.

18. This mathematician invented graph theory to solve the problem of the Seven Bridges of Königsberg. Their identity states that $e^{i\pi} + 1 = 0$. This mathematician follows the word "Project" to name a website dedicated to a series of computational problems intended to be solved using programming.